

ECON 500a  
General Equilibrium and Welfare Economics  
Edgeworth Box Economy

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# Outline: Static Exchange Economies

1. Edgeworth Box Economy
  2. Static Exchange Economy
  3. Efficiency and Welfare
  4. Microfounding Competition
  5. Competitive Equilibrium
- ▶ Readings
    - ▶ MWG: 15.B, 16.B, 16.E, 16.F, 22.B, 22.C, 22.E
    - ▶ Kreps: 14.3

# What is an Economy?

- ▶ General equilibrium studies economies
- ▶ An *economy* is a model of how economic activity (consumption, exchange, and production) takes place  
Defining economies clearly is critical to communicate economic theory
- ▶ Economies can be
  - ▶ **Static:** economic activity takes place simultaneously
  - ▶ **Dynamic/Stochastic:** economy activity involves time or uncertainty

# What is an Economy?

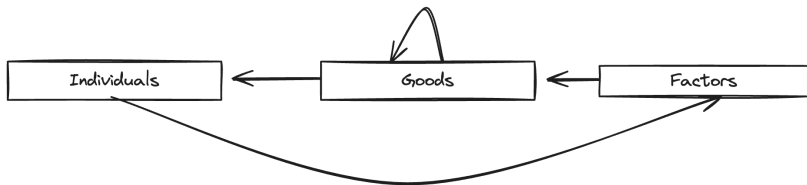
- ▶ Economies are populated by individuals
  - ▶ Economics is about people
- ▶ Individuals consume goods (and services) and supply factors

Terminology: goods = goods + services  
Classic GE: “commodities” = goods + services (+ factors)
- ▶ Production transforms factors and goods into other goods

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Terminology: goods = goods + services  
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- ▶ Production transforms factors and goods into other goods
- ▶ Physical Flow of Economic Activity:

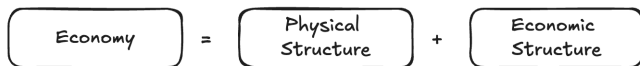


- ▶ We need prices, wages, income to define reverse arrows (but those are “competitive” notions)

# Endogenous & Exogenous Variables

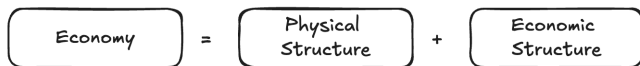
- ▶ Models consist of elements (variables, functions, functionals, etc.) that are
  - ▶ **Exogenous**: predetermined
  - ▶ **Endogenous**: determined by the model
- ▶ Distinction between endogenous & exogenous variables
  - ▶ For economy as a whole  
(e.g. prices are endogenous in general equilibrium)
  - ▶ For parts of the economy  
(e.g. prices are exogenous for a price-taker individual)
- ▶ **Comparative statics** study the response of endogenous variables when we change exogenous variables

# Defining an Economy



- ▶ Physical structure
  - ▶ Preferences
  - ▶ Technologies
  - ▶ Resource constraints

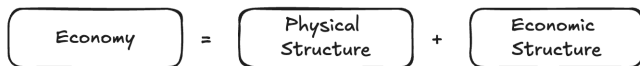
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- ▶ Economic structure in static economy:
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  - ii) how an individual behaves given the behavior of others (e.g.: price-taking behavior with linear budget constraints)
  - iii) how individuals interact (e.g.: equilibrium notion)
  - iv) how technologies are operated (e.g.: profit maximization by firms), etc.



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  - iv) how technologies are operated (e.g.: profit maximization by firms), etc.
- ▶ Economic structure in dynamic stochastic economies
  - i) the set of possible financial arrangements among individuals (e.g., are markets complete or incomplete)
  - ii) the ability or inability of individuals to keep promises (e.g., can individuals renege on their promises) and
  - iii) the information structure (e.g., who knows what when and how learning takes place)

## Edgeworth Box Economy: $I = J = 2$

- ▶ Foundational model of GE and WE  $\Rightarrow$  Pure Exchange Economy
- ▶  $I = 2$  individuals, indexed by  $i \in \mathcal{I} = \{1, 2\}$
- ▶  $J = 2$  goods, indexed by  $j \in \mathcal{J} = \{1, 2\}$ 
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- ▶ Resource constraint of good  $j$ :

$$\sum_i c^{ij} = \bar{y}^j \quad \text{where} \quad \bar{y}^j = \sum_i \bar{y}^{ij} > 0$$

- ▶ Ownership of endowments ( $\bar{y}^{ij} \geq 0$ ) matters for competitive equilibrium but not for planning solution

# Edgeworth Box Economy: $I = J = 2$

- ▶ Physical structure

$$V^1 = u^1(c^{11}, c^{12})$$

Preferences Individual 1

$$V^2 = u^2(c^{21}, c^{22})$$

Preferences Individual 2

$$c^{11} + c^{21} = \bar{y}^1$$

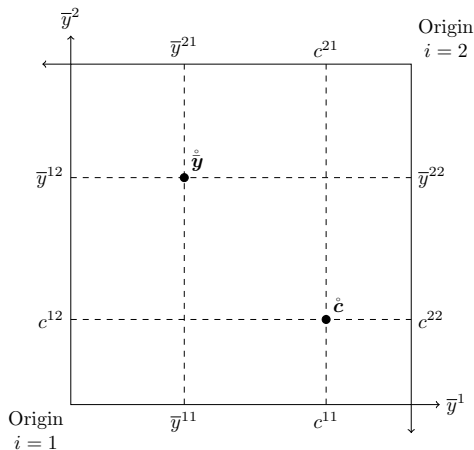
Resource Constraint Good 1

$$c^{12} + c^{22} = \bar{y}^2$$

Resource Constraint Good 2

- ▶ Where  $\bar{y}^1 = \bar{y}^{11} + \bar{y}^{21}$  and  $\bar{y}^2 = \bar{y}^{12} + \bar{y}^{22}$
- ▶ Non-negative consumption:  $c^{ij} \geq 0$ 
  - ▶ Variables in resource constraints must be non-negative
  - ▶ They reflect “sources” or “uses”

# Box Diagram



- ▶ Edgeworth (1881), Pareto (1906), and Bowley (1924)
  - ▶ Humphrey (1996) provides readable history

# Notation

- ▶ Allocation

$$\mathring{\mathbf{c}} = \{c^{11}, c^{21}, c^{12}, c^{22}\}$$

- ▶  $\mathbf{c}^1 = \{c^{11}, c^{12}\}$  and  $\mathbf{c}^2 = \{c^{21}, c^{22}\}$

- ▶ An allocation ( $\mathring{\mathbf{c}} \geq 0$ ) is *feasible* when resource constraints hold

- ▶ Endowments

$$\mathring{\mathbf{y}} = \{\bar{y}^{11}, \bar{y}^{21}, \bar{y}^{12}, \bar{y}^{22}\}$$

- ▶  $\bar{\mathbf{y}}^1 = \{\bar{y}^{11}, \bar{y}^{12}\}$  and  $\bar{\mathbf{y}}^2 = \{\bar{y}^{21}, \bar{y}^{22}\}$

- ▶ *Autarky* allocation is  $\mathring{\mathbf{c}} = \mathring{\mathbf{y}}$

## (Detour) On Notation

- ▶ Notation is challenging in GE
  - ▶ Many things to keep track of!
  - ▶ Unfortunately there is no standardized notation
- ▶ To ease transition between GE and applications  $\Rightarrow$  my notation follows Ljungqvist and Sargent (2018)
  - ▶ Consumption is denoted by  $c$  rather than  $x$  (as in MWG and traditional GE)
  - ▶ This course  $\Rightarrow$  **unified notation**
    - ▶ e.g. it'll become clear later why I use  $\check{c}$  rather than  $c$
- ▶ Bold variables are vectors or matrices
- ▶ Individual  $i$ 's consumption of good  $j$  at date  $t$  in history  $s^t$  is

$$c_t^{ij}(s^t)$$



## Remarks

1. (*Utilities vs. Preferences*) We could have defined preference relations or choice rules over  $\{c^{ij}\}_{j \in \mathcal{J}}$ 
  - ▶ See MWG Ch.1 for conditions: complete, transitive, continuous
2. (*Resource Constraints with Equality*) We could have written resource constraints with an inequality:

$$\sum_i c^{ij} \leq \bar{y}^j$$

- ▶ Equality involves little loss of generality  $\Rightarrow$  possible to introduce free-disposal technology for each good
3. (*Individuals vs. Types of Individuals*) At times, we take  $I = 2$  literally
    - ▶ In competitive economies  $\Rightarrow$  two *types* of individuals, with a continuum of each of the two types

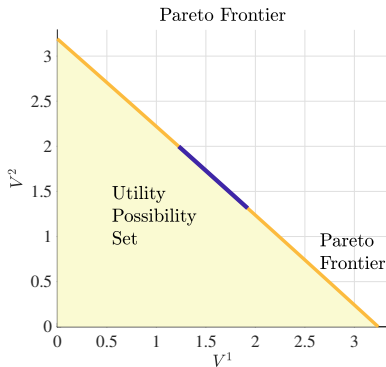
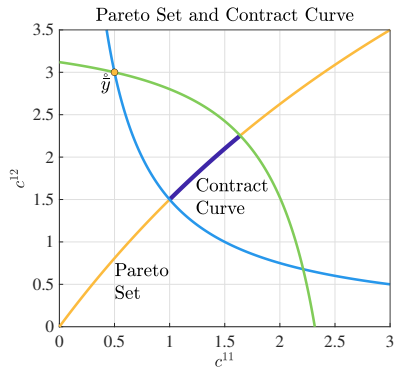
# Economic Structures

- ▶ Given the physical environment, which allocations would or should emerge endogenously?
  1. Planning problem
  2. Bargaining solution
  3. Competitive equilibrium

## Planning Problem: Definitions

- i) An allocation is *Pareto efficient* (or *Pareto optimal*) if there is no other feasible allocation such that all individuals are (weakly) better off, with at least one individual being strictly better off
- ii) The *Pareto set* is the set of Pareto efficient allocations
- iii) An allocation is *individually rational* for individual  $i$  if that individual prefers it to his/her endowment
- iv) The *contract curve* is the set of allocations that are Pareto efficient and individually rational for all individuals

# Planning Problem: Definitions



# Planning Problem

- ▶ How do we characterize Pareto efficient allocations, and therefore the Pareto set?
  - ▶ We solve planning problems
- ▶ The Pareto set corresponds to the set of allocations  $\mathring{c} = (c^{11}, c^{21}, c^{12}, c^{22})$  that solves the *planning problem*:

$$\max_{\mathring{c}} \alpha u^1(c^{11}, c^{12}) + (1 - \alpha) u^2(c^{21}, c^{22}),$$

where  $\alpha \in [0, 1]$  (Pareto weight), subject to

$$c^{11} + c^{21} = \bar{y}^1 \quad \text{Resource Constraint Good 1}$$

$$c^{12} + c^{22} = \bar{y}^2 \quad \text{Resource Constraint Good 2}$$

- ▶ Benevolent planners

# Planning Problem

1. Each value of  $\alpha$  indexes a different efficient allocation
  - ▶ Varying  $\alpha$  between 0 and 1 traces the Pareto set
2. (Interior) solutions satisfy resource constraints and the following two conditions:

$$\frac{\frac{\partial u^1}{\partial c^{11}}}{\frac{\partial u^2}{\partial c^{21}}} = \frac{\frac{\partial u^1}{\partial c^{12}}}{\frac{\partial u^2}{\partial c^{22}}} = \frac{1 - \alpha}{\alpha}$$

- ▶ Check that you know how to find them (we'll come back to this)
3. Planning problem can also be formulated as solving

$$\max_{\bar{c}} u^1(c^{11}, c^{12}) \quad \text{s.t.} \quad u^2(c^{21}, c^{22}) \geq \bar{u}$$

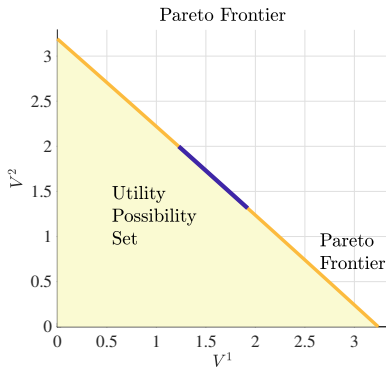
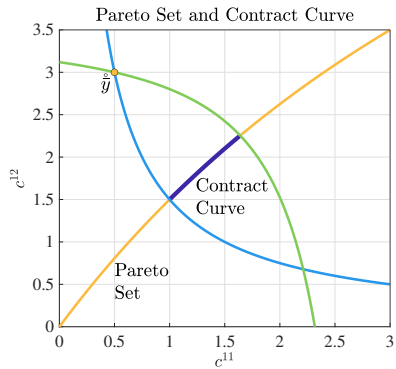
and resource constraints

- ▶ Varying the level of  $\bar{u}$  is equivalent to varying  $\alpha$
4. It seems reasonable that Pareto frontier allocations emerge endogenously once individuals communicate with each other
    - ▶ These allocations exhaust all gains from trade

# Pareto Frontier

- ▶ Space of utilities:
  - i) The *utility possibility set* corresponds to the set of attainable utility levels in an economy
  - ii) The *Pareto frontier* (or *utility possibility frontier*) corresponds to the set individual utilities associated with allocations in the Pareto set
- ▶ Utilities are only defined up to preference-preserving transformations (!!)
- ▶ If utility functions are concave (and it is possible to find a concave utility representation under minimal assumptions), then utility possibility set is convex: conditions in 16.E of MWG

# Pareto Frontier





# Axiomatic (Nash) Bargaining

- ▶ Solutions to the Pareto problem need not be individually rational
  - ▶ As  $\alpha \rightarrow 0$  or  $\alpha \rightarrow 1$ , one individual is worse off relative to their endowment of goods
- ▶ If individuals can unilaterally decide to consume their endowment, only individually rational allocations will be chosen  
⇒ **Contract curve**
- ▶ How do we characterize allocations in the contract curve?
  - ▶ We solve a Nash bargaining problem MWG 22.E
- ▶ *Nash bargaining* is an axiomatic bargaining procedure that selects allocations that satisfy a set of reasonable properties or axioms:
  - Pareto efficiency
  - individual rationality
  - independence of irrelevant alternatives  
If  $A$  is chosen over  $B$  in the choice set  $\{A, B\}$ , introducing a third option  $C$  must not result choosing  $B$  over  $A$
- ▶ **Remark:** here we take literally  $I = 2$ 
  - ▶ Two individuals, not two types of individuals

## Axiomatic (Nash) Bargaining

The contract curve corresponds to the set of allocations

$\hat{c} = (c^{11}, c^{12}, c^{21}, c^{22})$  characterized by solving the *Nash bargaining problem*:

$$\max_{\hat{c}} \left( u^1(c^{11}, c^{12}) - \underbrace{u^1(\bar{y}^{11}, \bar{y}^{12})}_{\text{disagreement}} \right)^\alpha \left( u^2(c^{21}, c^{22}) - \underbrace{u^2(\bar{y}^{21}, \bar{y}^{22})}_{\text{disagreement}} \right)^{1-\alpha},$$

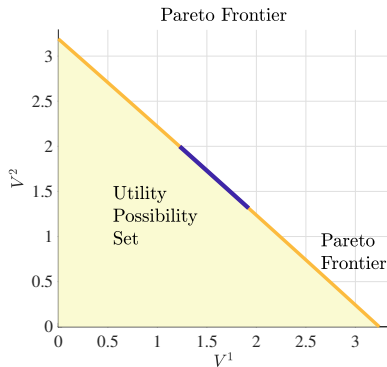
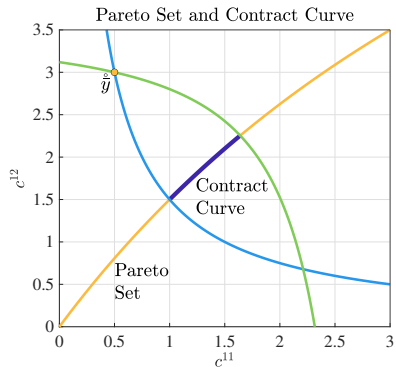
where  $\alpha \in [0, 1]$  (bargaining weight), subject to

$$c^{11} + c^{21} = \bar{y}^{11} + \bar{y}^{21} \quad \text{Resource Constraint Good 1}$$

$$c^{12} + c^{22} = \bar{y}^{12} + \bar{y}^{22} \quad \text{Resource Constraint Good 2}$$

- ▶ Contract curve allocations are good candidates to emerge endogenously because
  1. Exhaust all gains from trade
  2. Are robust to the possibility of an individual consuming his endowment in isolation
- ▶ Other axiomatic solutions exist, e.g. Kalai and Smorodinsky (1975)  
Textbook: Muthoo (1999)
  - ▶ Strategic bargaining (Rubinstein, 1982)

# Contract Curve



# Competitive/Walrasian Equilibrium

- ▶ Still many possible allocations in the contract curve
- ▶ Economic structure based on competitive, price-taking behavior

## Walrasian Model

- ▶ Individual choose consumption given prices, equilibrium is reached when prices are such that market clear
- ▶ Individual  $i$  solves

$$\max_{\mathbf{c}^i} u^i(c^{i1}, c^{i2}),$$

subject to a linear budget constraint given by

$$p^1 c^{i1} + p^2 c^{i2} = p^1 \bar{y}^{i1} + p^2 \bar{y}^{i2} \quad (1)$$

- ▶ (Marshallian) Demand

$$\mathbf{c}^i(\mathbf{p}, \bar{\mathbf{y}}^i)$$

# Equilibrium Definition

A *competitive equilibrium* is an allocation  $\mathbf{c} = (c^{11}, c^{12}, c^{21}, c^{22})$  and prices  $\mathbf{p} = (p^1, p^2)$  such that

- i) individuals chooses consumption to maximize utility subject to their budget constraint taking prices as given:

$$\mathbf{c}^i(\mathbf{p}, \bar{\mathbf{y}}^i) = \arg \max_{\mathbf{c}^i} u^i(c^{i1}, c^{i2}) \quad \text{s.t.} \quad p^1 c^{i1} + p^2 c^{i2} = p^1 \bar{y}^{i1} + p^2 \bar{y}^{i2},$$

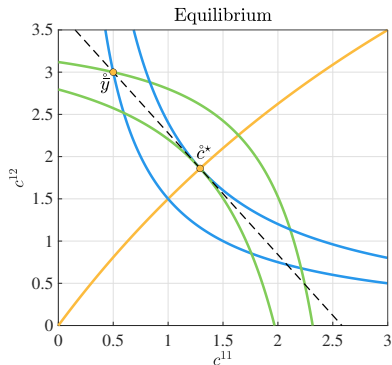
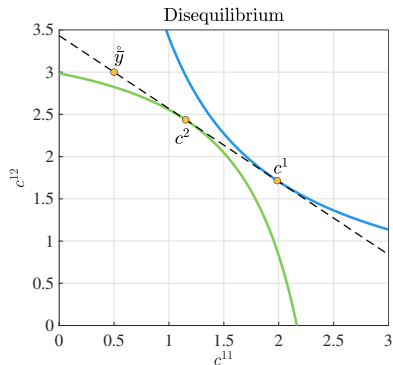
- ii) and markets clear, that is, resource constraints hold:

$$c^{11} + c^{21} = \bar{y}^{11} + \bar{y}^{21} \quad \text{Market Clearing Good 1}$$

$$c^{12} + c^{22} = \bar{y}^{12} + \bar{y}^{22} \quad \text{Market Clearing Good 2}$$

- ▶ First time we define an equilibrium. Key notion in this course.
  - ▶ “Allocation and prices such that individuals maximize and markets clear”
  - ▶ Always define your equilibrium notion!

# Equilibrium and Disequilibrium



- ▶ Disequilibrium: good 2 excess supply, good 1 excess demand
- ▶ Budget set(s) extend outside the box

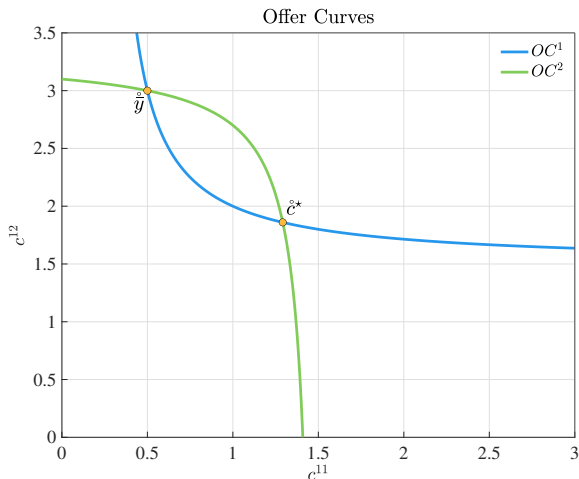
Budget Constraint:

$$c^{i2} = \underbrace{\frac{p^1}{p^2}}_{\text{slope}} (\bar{y}^{i1} - c^{i1}) + \bar{y}^{i2}$$

# Numeraire

- ▶ Individual demands  $\Rightarrow$  homogeneous of degree zero in prices
  - ▶ If  $\mathbf{p}$  is part of a competitive equilibrium  $\Rightarrow \alpha \mathbf{p}$  is an equilibrium too for  $\alpha > 0$
  - ▶ Trivial dimension of indeterminacy
- ▶ Choice of *numeraire*: good 1 as *numeraire*  $\Rightarrow p^1 = 1$ 
  - ▶ Or  $\sum_j p^j = 1$  (unit simplex)
- ▶ Walrasian model — at least in its basic form — is a theory of relative prices  $\frac{p^2}{p^1}$ 
  - ▶ No predictions for the aggregate price level  $\Rightarrow$  Monetary Theory Gale (1982, 1985), Woodford (2003), Williamson and Wright (2010), Starr (2013), Cochrane (2023)
  - ▶ “Hahn’s problem” (Hahn, 1965)

# Offer Curves



- ▶ The *offer curve* of individual  $i$  represents the demand function in the box diagram for different prices
  - ▶ Offer curve is tangent to indifference curve at the endowment
  - ▶ Intersections of offer curves (except at endowment)  $\Rightarrow$  equilibrium  
Why not at endowment? The prices that yield those offer curve points are different for each  $i$



# Excess Demand

- ▶ *Individual excess demand:*

$$z^{ij}(\mathbf{p}, \bar{\mathbf{y}}^i) = c^{ij}(\mathbf{p}, \bar{\mathbf{y}}^i) - \bar{y}^{ij}$$

- ▶  $z^{ij}(\mathbf{p}, \bar{\mathbf{y}}^i) > 0$ :  $i$  net buyer of good  $j$
- ▶  $z^{ij}(\mathbf{p}, \bar{\mathbf{y}}^i) < 0$ :  $i$  net seller of good  $j$
- ▶ *Aggregate excess demand:* sum of individual excess demands

$$z^j(\mathbf{p}; \overset{\circ}{\bar{\mathbf{y}}}) = \sum_i z^{ij}(\mathbf{p}, \bar{\mathbf{y}}^i) = \sum_i (c^{ij}(\mathbf{p}, \bar{\mathbf{y}}^i) - \bar{y}^{ij})$$

- ▶  $z^j(\mathbf{p}; \overset{\circ}{\bar{\mathbf{y}}}) > 0$ : aggregate excess demand of good  $j$
- ▶  $z^j(\mathbf{p}; \overset{\circ}{\bar{\mathbf{y}}}) < 0$ : aggregate excess supply of good  $j$

# Excess Demand

- ▶  $J$ -dimensional vectors:

$$z^i(\mathbf{p}, \bar{\mathbf{y}}^i) = \mathbf{c}^i(\mathbf{p}, \bar{\mathbf{y}}^i) - \bar{\mathbf{y}}^i$$

$$\mathbf{z}(\mathbf{p}; \overset{\circ}{\mathbf{y}}) = \sum_i z^i(\mathbf{p}, \bar{\mathbf{y}}^i) = \sum_i (\mathbf{c}^i(\mathbf{p}, \bar{\mathbf{y}}^i) - \bar{\mathbf{y}}^i)$$

- ▶ Competitive equilibrium prices  $\mathbf{p}^*$  solve the system

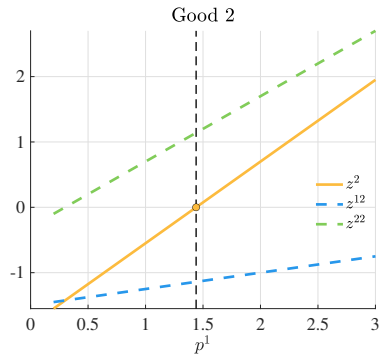
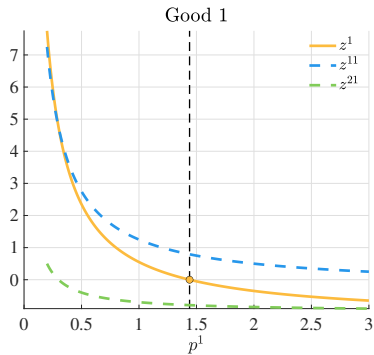
$$\mathbf{z}(\mathbf{p}^*; \overset{\circ}{\mathbf{y}}) = 0$$

- ▶ Aggregate excess demand map  $\mathbf{z}(\mathbf{p}; \overset{\circ}{\mathbf{y}})$  summarizes the positive properties of competitive equilibria
  - ▶ We'll study its properties later in the course
- ▶ Competitive equilibrium allocations given by

$$\boxed{\mathbf{c}^{i^*}(\overset{\circ}{\mathbf{y}}) = \mathbf{c}^i(\mathbf{p}^*, \bar{\mathbf{y}}^i)} \quad \text{where } \mathbf{p}^* \text{ solves } \boxed{\mathbf{z}(\mathbf{p}^*; \overset{\circ}{\mathbf{y}}) = 0}$$

# Excess Demand

Excess Demand: Cobb-Douglas



# Walras' Law

▶ Walras' Law:

*“if all but one markets in a competitive economy are in equilibrium, then the last market must also be in equilibrium”*

▶ Walras' Law is a property of the aggregate excess demand function

▶ Proof: aggregating all individual budget constraints

Individual budget constraints can be written as  $\sum_j p^j (c^{ij} - \bar{y}^{ij}) = 0, \forall i$

$$\sum_i \sum_j p^j (c^{ij} - \bar{y}^{ij}) = \sum_i \sum_j p^j z^{ij} (\mathbf{p}, \bar{\mathbf{y}}^i) = \boxed{\mathbf{p} \cdot \mathbf{z} (\mathbf{p}; \overset{\circ}{\mathbf{y}}) = 0}$$

▶ In Edgeworth box economy:

$$p^1 (c^{11} + c^{21} - \bar{y}^{11} - \bar{y}^{21}) + p^2 (c^{12} + c^{22} - \bar{y}^{12} - \bar{y}^{22}) = 0$$

▶ If  $c^{11} + c^{21} = \bar{y}^{11} + \bar{y}^{21} = 0$ , then  $c^{12} + c^{22} = \bar{y}^{12} + \bar{y}^{22} = 0$ .

▶ We can drop one of the conditions in  $\mathbf{z} (\mathbf{p}^*; \overset{\circ}{\mathbf{y}}) = 0$

# Walras' Law

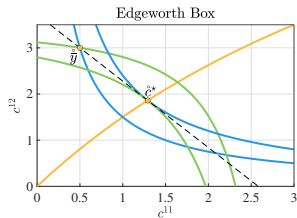
- ▶ An implication of Walras' Law is that  
*“if there is aggregate excess demand in one market, there must be another market with aggregate excess supply”*
- ▶ If  $c^{11} + c^{21} - \bar{y}^{11} - \bar{y}^{21} > 0$ , then  $c^{12} + c^{22} - \bar{y}^{12} - \bar{y}^{22} < 0$ 
  - ▶ “Aggregate expenditures are constrained by aggregate income”
- ▶ **Remark:** Walras' law exclusively uses linearity of budget constraints
  - ▶ It does not require individual optimality or price-taking behavior
  - ▶ Walras' law may also hold in not perfectly competitive models

# Graphical Representation

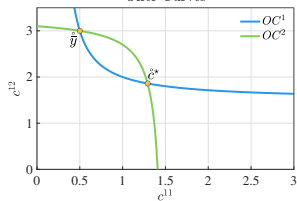
- i) [*Box diagram*] The top left diagram shows the box diagram with the endowments, the Pareto set, the contract curve, and the competitive equilibrium. It displays indifference curves in autarky and at the competitive equilibrium
- ii) [*Box diagram with offer curves*] The top right diagram also shows the box diagram but it only displays the offer curves. The (non-autarky) allocations at which the offer curves intersect characterize the set of competitive equilibria
- iii) [*Utility diagram*] The bottom left diagram shows the utility possibility frontier and the Pareto frontier, it also displays the values of utilities at the autarky allocation and the competitive equilibrium
- iv) [*Excess demand diagram*] The bottom right diagram shows aggregate excess demand as a function of  $p = p^1/p^2$

# Cobb Douglas

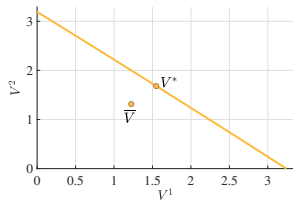
Edgeworth Box with Cobb-Douglas Preferences



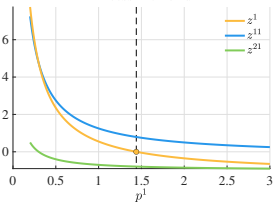
Offer Curves



Pareto Frontier



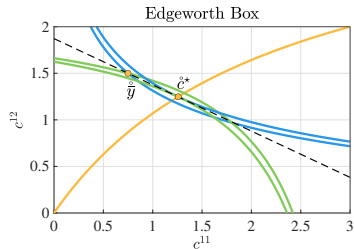
Excess Demand



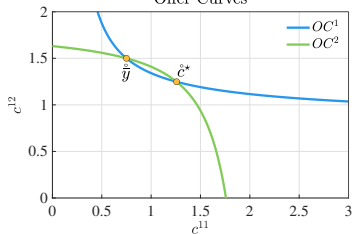
►  $V^i = (c^{i1})^{\alpha^i} (c^{i2})^{1-\alpha^i}$

# Isoelastic (CES)

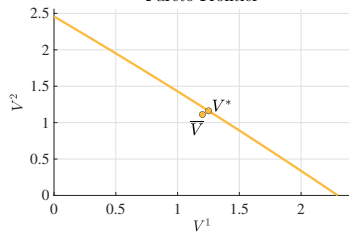
Edgeworth Box with CES Preferences



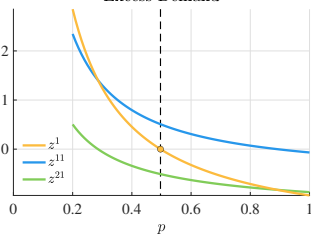
Offer Curves



Pareto Frontier



Excess Demand

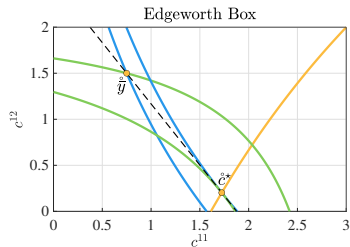


$$\blacktriangleright V^i = \left[ \alpha^i (c^{i1})^{\frac{\epsilon-1}{\epsilon}} + (1 - \alpha^i) (c^{i2})^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

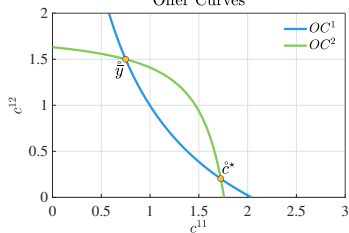


# Stone-Geary CES

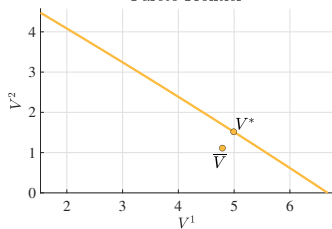
Edgeworth Box with Stone-Geary Preferences



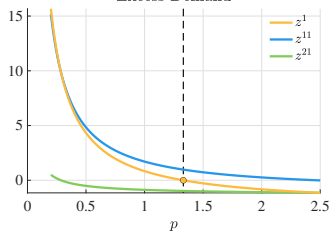
Offer Curves



Pareto Frontier



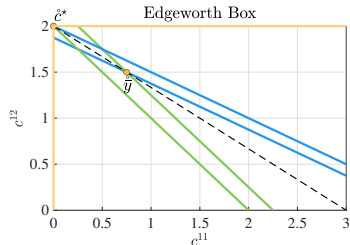
Excess Demand



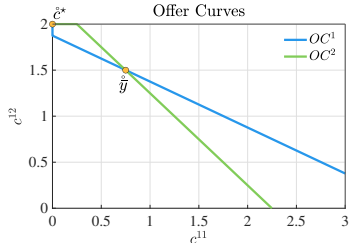
$$\blacktriangleright V^i = \left[ \alpha^i (c^{i1} - \bar{c}^{i1})^{\frac{\epsilon-1}{\epsilon}} + (1 - \alpha^i) (c^{i2} - \bar{c}^{i2})^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

# Perfect Substitutes

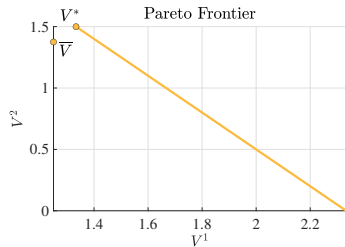
Edgeworth Box with Perfect-Substitutes



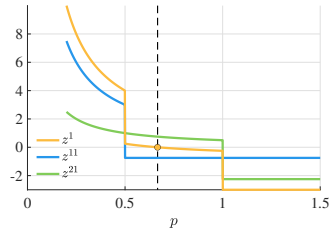
Offer Curves



Pareto Frontier



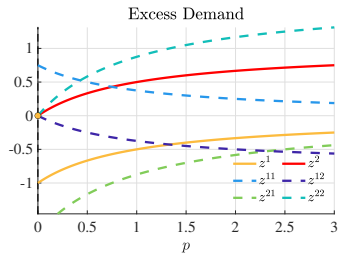
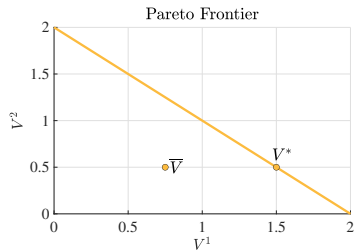
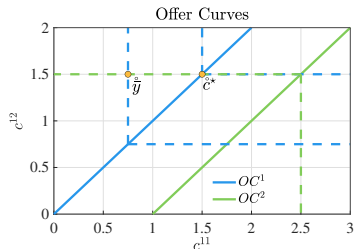
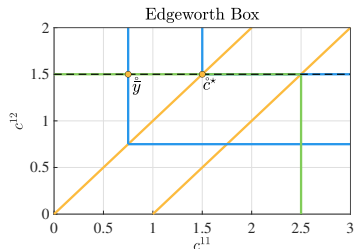
Excess Demand



►  $V^i = \alpha^i c^{i1} + (1 - \alpha^i) c^{i2}$ .

# Perfect Complements

Edgeworth Box with Perfect-Complements



►  $V^i = \min \{c^{i1}, c^{i2}\}$

# Final Observations

- i) Box diagram is an incredibly powerful tool
  - ▶ It can illustrate almost all GE and WE phenomena in static exchange economies
  - ▶ Less useful to think about production and dynamic stochastic economies
- ii) Many papers (including some of mine!) do not distinguish between physical and economic structure
  - ▶ I encourage you to always be clear about the physical structure of your economy and to first characterize the planning problem
- iii) There are three types of “consumption” Edgeworth boxes
  - ▶ This section corresponds to a “static” Edgeworth box economy
  - ▶ We explore “dynamic” and “stochastic” Edgeworth box economies in Block III
- iv) There are also “factor” Edgeworth boxes, which use the box diagram with two factors of production instead of two goods
  - ▶ See Helpman and Krugman (1985) or Bhagwati, Panagariya, and Srinivasan (1998)

# Edgeworth Box

*In a canvas of possibilities, two souls trade,  
Within the box of Edgeworth, their desires laid.  
Each curve is a whisper of want, a silent plea,  
For goods to shift hands in perfect harmony.*

*With endowments they begin, their claims set clear,  
Seeking gains through trade, as they draw near.  
Where curves embrace, an equilibrium's born,  
No more to gain, no longer forlorn.*

*This box, a world where balance reigns,  
No bids are shouted, no voice strains.  
Here, through unseen hands and prices set to stake,  
Efficiency thrives, though fairness may shake.*

*ChatGPT (with some help) 10/17/2024*

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