ECON 500a
General Equilibrium and Welfare Economics
Microfounding Competition

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Outline: Static Exchange Economies

1. Edgeworth Box Economy
2. Static Exchange Economy
3. Efficiency and Welfare
4. Microfounding Competition
5. Competitive Equilibrium

- Readings
  - MWG: 18.B

Motivation

- So far, we have assumed linear pricing and price-taking $\rightarrow$ Why?
  - This does not really make sense with $I = 2$ or I small
- Will the competitive allocations (and prices) emerge endogenously when people behave strategically?
  - Under which conditions?
- Strategic Foundations of Walrasian Equilibrium
  1. Cooperative game theory
     - Core Equivalence/Convergence Theorem
  2. Non-cooperative game theory $\rightarrow$ Not this course

Edgeworth Had Really Good Intuition

- Edgeworth (1881): "Mathematical Psychics"

  "As the number of participants increases indefinitely, the final settlements tend to converge towards a unique result, in which no participant has an incentive to vary his terms. Thus, in the limit of an infinitely large number of bargainers, the monopoly element vanishes, and the results of competition are obtained."

- Also

  > *"The range of indeterminateness of the final settlement decreases as the number of bargainers increases."*

- "Edgeworth's limit theorem"
- My treatment follows Debreu and Scarf (1963), same as MWG

Environment and Definitions

- Static exchange economy
  - $i \in \mathcal{I} = \{1, \ldots, I\}$ and $j \in \mathcal{J} = \{1, \ldots, J\}$
  - I taken literally here
- Continuous, convex, and strictly monotone preferences
- Definition: A coalition $\mathcal{N} \subseteq \mathcal{I}$ *blocks* the feasible allocation $\mathring{\boldsymbol{c}}^{\star}$ if there is another allocation $\mathring{\boldsymbol{c}}$ such that:
  1. Everyone in the coalition is better off

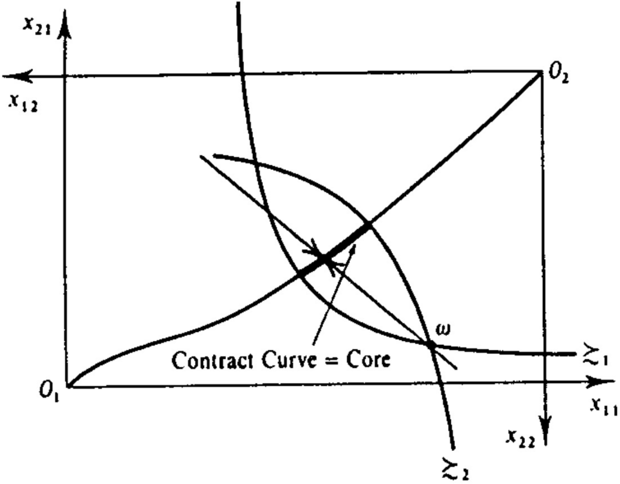
$$u^n\left(\boldsymbol{c}^n\right) > u^n\left(\boldsymbol{c}^{n\star}\right), \quad \forall n \in \mathcal{N}$$

  2. It is feasible using the endowments of the coalition

$$\sum_{n \in \mathcal{N}} c^{nj} \leq \sum_{n \in \mathcal{N}} \bar{y}^{nj}, \quad \forall j \in \mathcal{J}$$

- Definition: The *core* is the set of allocations that cannot be blocked

Core in Edgeworth Box



- In Edgeworth box economies: $\boxed{\text{Core} \iff \text{Contract curve}}$
  - Allocations outside of Pareto set dominated by coalitions of $I = 2$
  - Allocations not individual rational dominated by coalitions of $I = 1$

Core Equivalence Theorem

- **Core Equivalence Theorem**: combination of two results
  1. Competitive equilibrium allocations are in the core
  2. As the number of individuals increases ($I \to \infty$), the core shrinks to the set of CE

Core Equivalence Theorem: CE in core

Result #1: Competitive equilibrium allocations are in the core

- Proof: same logic as proof #1 of First Welfare Theorem
  1. Consider a competitive equilibrium (CE) given by $\mathring{\boldsymbol{c}}^\star$ and $\boldsymbol{p}^\star$
  2. Consider a coalition $\mathcal{N} \subseteq \mathcal{I}$ and an allocation $\mathring{\boldsymbol{c}}$ such that (with inequality for at least one)

$$u^n\left(\boldsymbol{c}^n\right) \geq u^n\left(\boldsymbol{c}^{n\star}\right), \quad \forall n \in \mathcal{N}$$

     - If this allocation were feasible, it would block the CE
  3. At prices $\boldsymbol{p}^\star$: $\sum_j p^{j\star} c^{nj} > \sum_j p^{j\star} \bar{y}^{nj}$ for one individual and $\sum_j p^{j\star} c^{nj} \geq \sum_j p^{j\star} \bar{y}^{nj}$ for the others
     - Aggregating among coalition members $n \in \mathcal{N}$

$$\sum_{n\in\mathcal{N}} \sum_j p^{j\star} c^{nj} > \sum_{n\in\mathcal{N}} \sum_j p^{j\star} \bar{y}^{nj} \Rightarrow \sum_j p^{j\star} \left(\sum_{n\in\mathcal{N}} c^{nj} - \sum_{n\in\mathcal{N}} \bar{y}^{nj} \right) > 0$$

     - But "coalition resource constraint" is $\sum_{n\in\mathcal{N}} c^{nj} = \sum_{n\in\mathcal{N}} \bar{y}^{nj}$
     - Proposed blocking allocation not feasible
  4. The <u>CE cannot be blocked</u>

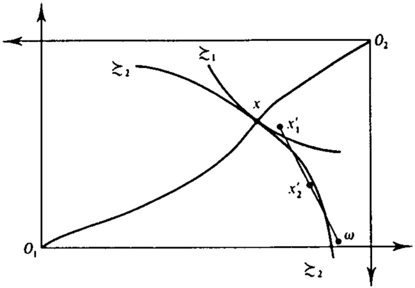
Towards a Converse

- Can all allocations in the core be supported as a competitive equilibrium (without transfers)?
  - Obviously not
- Take Cobb-Douglas preferences with $I = 2$
  - Unique CE
  - Core (Contract Curve) has many allocations

Replica Economies

- Replica economy: $I \times M$ individuals
  Debreu and Scarf (1963)
- $i \in \mathcal{I} = \{1, \ldots, I\}$ types
- $m \in \mathcal{M} = \{1, \ldots, M\}$ individuals per type
- Interesting limit
  - Fix number of types
  - Increase number of individuals per type: $\boxed{M \to \infty}$

Illustration Economies



- First assume $I = 2$ and $M = 1$ (Edgeworth box)
  - Initial allocation (x) is in the core but it is *not* a CE
- Why? Let's assume that $M = 2 \Rightarrow$ Allocation x' blocks x
  - $x'_1 - \omega_1 + 2\,(x'_2 - \omega_2) = 0$ or $c^{1j} - \bar{y}^{1j} + 2\left(c^{2j} - \bar{y}^{2j}\right) = 0, \forall j$
  - Type-1 individuals are favored in initial allocation relative to CE
  - Two type-2 individuals form a coalition with one type-1 individual
  - Coalition of one agent of type-1 and two agents of type 2
- As number of individuals grows $(M \to \infty) \to$ Core shrinks

Core Equivalence Theorem

- Assume "equal treatment" (without proof): every individual of type i must have the same allocation in a CE
  - Lengthy proof in MWG
  - Idea: build coalition of the worse treated individuals of each type
- Result #2: (Core convergence) If the feasible type allocation $\mathring{\boldsymbol{c}}^{\star}$ (IJ-dimensional) is in the core for all $M = 1, 2, \ldots,$ then $\mathring{\boldsymbol{c}}^{\star}$ is a CE
  - Proof: next slide

The core "shrinks" as M grows

Proof

- Proof strategy
  - Suppose that $\mathring{\boldsymbol{c}}^\star$ (IJ-dimensional) is a feasible type-allocation, but not a CE
  - Then it can be blocked
- Proof:
  1. If $\mathring{\boldsymbol{c}}^\star$ not Pareto Optimal, coalition of everyone can block it
     - Therefore, assume that $\mathring{\boldsymbol{c}}^\star$ is Pareto optimal
  2. Since $\mathring{\boldsymbol{c}}^\star$ is Pareto Optimal, it can be supported as a CE with transfers (we denote prices as $\boldsymbol{p}^\star$)
     - Via second welfare theorem
  3. If $\mathring{\boldsymbol{c}}^\star$ is not a CE, then some type (assume type-1) is favored relative to CE
     - So $\sum_j p^{j\star}\left(c^{1j\star} - \bar{y}^{1j}\right) > 0$

Core Equivalence Theorem: Proof

4. If M is large enough, there is a coalition ($\mathcal{N}$, with $n \in \mathcal{N} = \{1, \ldots, N\}$) of all types with $M - 1$ individuals of type 1 to exclude one type-1 individual
   - Number of individuals in the coalition: $N = \underbrace{M - 1}_{\text{type 1}} + \underbrace{M(I - 1)}_{\text{others types}}$
   - Coalition pattern has Bertrand competition flavor
5. Proposed allocation change

   Remember: $\boldsymbol{c}^n = \left\{c^{n1}, \ldots, c^{nJ}\right\}$ and $\bar{\boldsymbol{y}}^n = \left\{\bar{y}^{n1}, \ldots, \bar{y}^{nJ}\right\}$

$$\Delta \boldsymbol{c}^n = \boldsymbol{c}^{n\prime} - \boldsymbol{c}^n = \frac{1}{N}\left(\boldsymbol{c}^1 - \bar{\boldsymbol{y}}^1\right)$$

   - Distribute symmetrically excluded type-1 net trade $(\boldsymbol{c}^1 - \bar{\boldsymbol{y}}^1)$
   - Negative entries of the vector $\boldsymbol{c}^1 - \bar{\boldsymbol{y}}^1$ make things hard
   - But as M (and N) grows: $\lim_{M\to\infty} \Delta \boldsymbol{c}^n = \frac{d\boldsymbol{c}^n}{d\theta} \to$ marginal argument

Core Equivalence Theorem: Proof

6. Does everyone in the coalition gain? Yes!
   - For any individual in the coalition we have formed ($n \in \mathcal{N}$)

$$\frac{\frac{dV^n}{d\theta}}{\lambda^n} = \sum_j \frac{\frac{\partial u^n}{\partial c^{nj}}}{\lambda^n} \frac{dc^{nj}}{d\theta} = \sum_j p^j \frac{dc^{nj}}{d\theta}$$

   - But since type-1 was favored

$$\sum_j p^j \frac{dc^{nj}}{d\theta} = \boldsymbol{p}\Delta\boldsymbol{c}^n = \frac{1}{N}\sum_j p^{j\star}\left(c^{1j\star} - \bar{y}^{1j}\right) > 0$$

   - Everyone in the coalition is happy at the margin!
   - Intuition: the excluded type-1 was consuming too much overall
     - The value of his net consumption was too high
     - The others kick him out and split that extra value!

As $M, N \to \infty$, the only allocations that cannot be blocked are CE

Core Equivalence Theorem: General Results

- Much more general proofs exist
  - Aumann (1964): continuum of agents
  - Anderson (1978): nonstandard analysis
  - These proofs are valid under minimal regularity assumptions
- The "core" used widely in matching (*discrete* economies)
  - Shapley and Scarf (1974) → houses → Top trading cycles
    - Recent survey: Afacan, Hu, and Li (2024)
  - Gale and Shapley (1962) → Deferred acceptance algorithm
  - Shapley and Shubik (1971)
  - Roth and Sotomayor (1992)
  - Roth (2015) → organ donation, college admissions, and job placements

Non-Cooperative Foundations

- For any non-competitive economic structure, it is always possible to ask whether a limit delivers the competitive outcome
- Examples:
  1. Bertrand yields competitive outcome
  2. Cournot converges to competitive outcome as number of firms grow
  3. Shapley and Shubik (1977) analyze market games and how they converge to a competitive equilibrium
  4. Monopolistic competition models yield a competitive outcome as individual preferences are perfectly elastic, but not as fixed costs go to zero (Córdoba and Liu, 2023)
  5. Gale (2000) $\rightarrow$ strategic and game-theoretic foundation to competitive equilibrium applying dynamic matching and bargaining models

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