ECON 500a
General Equilibrium and Welfare Economics
Elementary Static Production Economies

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Course Outline

- Block 1: Static Exchange Economies
- Block 2: Static Production Economies
- Block 3: Dynamic Stochastic Economies

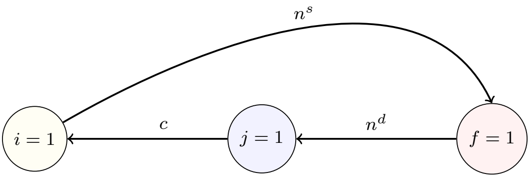
Outline: Static Production Economies

1. Elementary Static Production Economies
   i) Robinson Crusoe
   ii) Horizontal Production
   iii) Vertical Production
   iv) Quasilinear
2. General Static Production Economies
3. Efficiency and Welfare
4. Applications

- Readings
  - MWG: 5.C, 5.D, 5.G, 15.B

Robinson Crusoe Economy: $I = J = F = 1$

- Most elementary production model
  - $I = 1$ individual, indexed by $i \in \mathcal{I} = \{1\}$
  - $J = 1$ good, indexed by $j \in \mathcal{J} = \{1\}$
  - $F = 1$ factor, indexed by $f \in \mathcal{F} = \{1\}$
- Consistent notation $\Rightarrow$ But we drop degenerate dimensions
  - Instead of V^i and $c^{ij} \rightarrow V$ and c
  - In general: $n^{if,s}$ and $n^{jf,d}$



- Useful to think about **Aggregate Factor Efficiency**
  - If the aggregate amount of hours supplied efficient?
  - Is the economy in a recession?

Robinson Crusoe Economy: Physical Structure

- Preferences

$$V = u\left(c, n^s\right)$$

- Technology

$$y = G\left(n^d\right)$$

  - CRS: $G\left(n^d\right) = an^d$, where $a > 0$
  - DRS: $G\left(n^d\right) = a\left(n^d\right)^\eta$, where $a > 0$ and $0 < \eta < 1$
- Good resource constraint

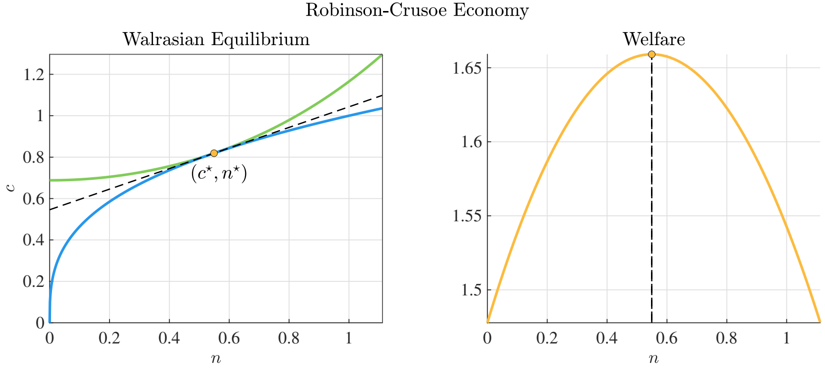
$$y = c$$

- Factor resource constraint

$$n^s = n^d$$

- An *allocation* is $\left\{c, y, n^s, n^d\right\}$

Robinson Crusoe



Planning Problem

$$\max_{\{c,y,n^s,n^d\}} u\left(c, n^s\right)$$

subject to

$$y = G\left(n^d\right), \qquad c = y, \qquad n^s = n^d$$

- 4 choices − 3 constraints = 1 degree of freedom $\rightarrow n^s$

$$\max_{n^s} u\left(G\left(n^s\right), n^s\right)$$

- Solution:

$$\underbrace{\frac{\partial u}{\partial c}\frac{\partial G}{\partial n^d}}_{\substack{\text{Mg. Benefit of} \\ \text{increasing } n^s}} = \underbrace{-\frac{\partial u}{\partial n^s}}_{\substack{\text{Mg. Cost of} \\ \text{increasing } n^s}} \implies n^{s\star}$$

- Aggregate factor efficiency: is Robinson working the right amount of hours?
  - $MWP = MRS$
- If a single individual operates technology $\rightarrow$ We expect this allocation
  - Robinson Crusoe alone in the island!

Competitive Equilibrium: Definition

- A *competitive equilibrium* is an allocation $\left\{c, y, n^s, n^d\right\}$, a price p, and a wage w, such that
  i) individual chooses consumption and factor supply to maximize utility subject to the budget constraint taking prices as given

$$\max_{\{c,n^s\}} u\left(c, n^s\right) \quad \text{s.t.} \quad pc = wn^s + \pi$$

  i) technology is operated to maximize profits choosing factor use taking prices as given

$$\pi = \max_{n^d} \left\{py - wn^d\right\} \quad \text{where} \quad y = G\left(n^d\right)$$

  iii) and markets clear, that is, resource constraints hold:

$$c = y \qquad \text{Resource Constraint Good}$$
$$n^s = n^d \qquad \text{Resource Constraint Factor}$$

- **Remark**: very different notion compared to planning problem!
  - Many identical "Crusoe's" consuming and producing

Competitive Equilibrium: Characterization

1. Individual problem: $\max_{\{c,n^s\}} u(c, n^s) \quad \text{s.t.} \quad pc = wn^s + \pi$
   - Optimality: $\frac{\partial u}{\partial c}\frac{w}{p} = -\frac{\partial u}{\partial n^s} \implies \frac{-\frac{\partial u}{\partial n^s}}{\frac{\partial u}{\partial c}} = \frac{w}{p}$
   - MRS between c and n^s equalized with real wage
2. Technology: $\pi = \max_{n^d}\{py - wn^d\} \quad \text{where} \quad y = G(n^d)$
   - Optimality: $\to$ "mg. cost pricing"

$$\underbrace{p\frac{\partial G}{\partial n^d}}_{\substack{\text{Mg. Revenue}\\ \text{Product}}} = \underbrace{w}_{\substack{\text{Cost of}\\ \text{Factor (wage)}}} \implies \frac{\partial G}{\partial n^d} = \frac{w}{p}$$

   - DRS: $\pi > 0$ and interior solution
   - CRS: $\pi = (pa - w)\, n^d = 0$, so $a = \frac{w}{p}$, with $\pi = 0$

Competitive Equilibrium: Characterization

- Combining both:

$$\frac{\partial G}{\partial n^d} = \frac{w}{p} = \frac{-\frac{\partial u}{\partial n^s}}{\frac{\partial u}{\partial c}} \implies \boxed{\frac{\partial u}{\partial c}\frac{\partial G}{\partial n^d} = -\frac{\partial u}{\partial n^s}}$$

  - Same as planning solution!
  - First Welfare Theorem: CE is Pareto Optimal

Remarks

1. *(What is a firm? What is its objective?)* In production economies, technologies are operated by *"firms"*
   - Firms maximize profit → Why?
     - Easy to justify in static competitive models
   - Objective of the firm → Unclear!
     - Dynamic stochastic incomplete market models
     - Imperfectly competitive models
     - Hart (1995)
   - For brevity, I may use the word "firm" rather than "technology-operating agent"

Remarks

2. *(Walras' Law)* With production, Walras' law follows from aggregating i) budget constraints and ii) profits
   - Proof:

$$pc = wn^s + py - wn^d \implies p(c - y) = w\left(n^s - n^d\right),$$

     so if $c = y$, then $n^s = n^d$ and vice versa
   - Similarly, if $pc > py$, then $wn^s > wn^d$, and vice versa
3. *(Numeraire)* Walrasian model remains a model of relative prices, also with production
   - With $J + F = 2$, we get to normalize one price or wage
   - Model predicts only $\frac{w}{p}$

Equivalence between CRS and DRS

- **Claim**: Equivalence between (McKenzie, 1959)
  - i) CRS technology with F variable factors
  - ii) DRS technology with $F-1$ variable factors and 1 fixed factor
- Modified Robinson Crusoe economy:
  - $V = u\left(c, n^{1,s}\right)$, $c = y$, $n^{1,s} = n^{1,d}$, $\bar{n}^{2,s} = n^{2,d}$
- CRS Technology $F = 2$:

$$y = G\left(n^{1,d}, n^{2,d}\right) = A\left(n^{1,d}\right)^{\eta}\left(n^{2,d}\right)^{1-\eta}$$

  - Factor 2 appears as fixed endowment: $\bar{n}^{2,s} = n^{2,d}$
- Euler's Theorem for homogeneous functions

$$G\left(n^{1,d}, n^{2,d}\right) = \frac{\partial G}{\partial n^{1,d}} n^{1,d} + \frac{\partial G}{\partial n^{2,d}} n^{2,d}$$

Equivalence between CRS and DRS

$$y = G\left(n^{1,d}, n^{2,d}\right) = A\left(n^{1,d}\right)^{\eta}\left(n^{2,d}\right)^{1-\eta}$$

1. Planning solution
   - Unchanged: $y = G\left(n^{1,d}\right) = \tilde{A}\left(n^{1,d}\right)^{\eta}$ where $\tilde{A} = A\left(\bar{n}^{2,s}\right)^{1-\eta}$
2. Firm owns factor 2 in a CE

$$\pi = pG\left(n^{1,d}, n^{2,d}\right) - w^1 n^{1,d} = \underbrace{\left(p\frac{\partial G}{\partial n^{1,d}} - w^1\right)}_{=0 \text{ (optimality)}} n^{1,d} + p\frac{\partial G}{\partial n^{2,d}}\bar{n}^{2,s} > 0$$

   - $pc = w^1 n^{1,s} + \pi$ where $\boxed{\pi = p\frac{\partial G}{\partial n^{2,d}}\bar{n}^{2,s}}$
3. Individual owns factor 2 in a CE

$$\begin{aligned}\pi &= pG\left(n^{1,d}, n^{2,d}\right) - w^1 n^{1,d} - w^2 n^{2,d} \\ &= \underbrace{\left(p\frac{\partial G}{\partial n^{1,d}} - w^1\right)}_{=0} n^{1,d} + \underbrace{\left(p\frac{\partial G}{\partial n^{2,d}} - w^2\right)}_{=0} n^{2,d} = 0\end{aligned}$$

   - $pc = w^1 n^{1,s} + \boxed{w^2 \bar{n}^{2,s}}$ where $\boxed{w^2 = p\frac{\partial G}{\partial n^{2,d}}}$

CE and Profits

- Competitive equilibrium can feature $\pi > 0$
  - Return to fixed factor
- McKenzie (1959): CRS without loss
  - As long as we allow for enough fixed factors

Robinson Crusoe

Alone on an island, Crusoe wakes each day,
With scarce resources, yet dreams to weigh.
His choices define the bounds of his fate,
To fish or to farm, to build or to wait.

The solitary man, both worker and king,
Production and consumption, in balance they swing.
No markets to guide, no prices to call,
Only his labor, his time, and nature's thrall.

In this quiet world, a simple dance unfolds,
Effort for survival, in stories untold.
Yet even here, an invisible hand may sway,
For trade with himself sets his path each day.

ChatGPT 10/17/2024 (with some help)

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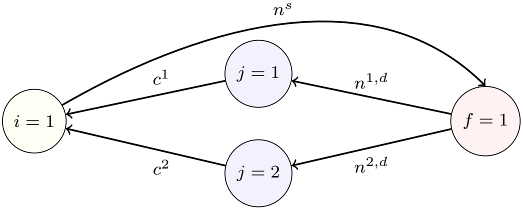
   iii) Vertical Production
   iv) Quasilinear
2. General Static Production Economies
   - Efficiency and Welfare
   - Competition
3. Applications

Horizontal Production Economy: $I = F = 1$, $J = 2$

- (Second) Most elementary production model
  - $I = 1$ individual, indexed by $i \in \mathcal{I} = \{1\}$
  - $J > 1$ goods, indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$
  - $F = 1$ factor, indexed by $f \in \mathcal{F} = \{1\}$
- Factor can be fixed or elastically supplied



- Useful to think about Misallocation
  - Cross-sectional factor use efficiency
  - Are factors efficiently allocated?

Horizontal Production Economy: Physical Structure

- Preferences

$$V = u\left(\left\{c^j\right\}_{j\in\mathcal{J}}, n^s\right)$$

- Technologies

$$y^j = G^j\left(n^{j,d}\right)$$

- Good resource constraints

$$y^j = c^j$$

- Factor resource constraint

$$n^s + \bar{n}^s = n^d$$

- An *allocation* $(J = 2)$ is $\left\{c^1, c^2, y^1, y^2, n^{1,d}, n^{2,d}, n^s\right\}$

Planning Problem

$$\max_{\{c^1, c^2, y^1, y^2, n^{1,d}, n^{2,d}\}} u\left(c^1, c^2, n^s\right)$$

subject to

$y^1 = G^1\left(n^{1,d}\right)$	Technology Good 1
$y^2 = G^2\left(n^{2,d}\right)$	Technology Good 2
$y^1 = c^1$	Resource Constraint Good 1
$y^2 = c^2$	Resource Constraint Good 2
$\bar{n}^s = n^{1,d} + n^{2,d}$	Resource Constraint Factor

- Fixed factor: 1 degree of freedom
- Elastic factor: 2 degrees of freedom

Planning Problem: Fixed Factor

$$\max_{\{n^{1,d},n^{2,d}\}} u\left(G^1\left(n^{1,d}\right), G^2\left(n^{2,d}\right)\right) \quad s.t. \quad \bar{n}^s = n^{1,d} + n^{2,d}$$

- In terms of factor use shares: $\chi_n^{1,d} = \frac{n^{1,d}}{n^d}$

$$\max_{\chi_n^{1,d}} u\left(G^1\left(\chi_n^{1,d}\bar{n}^s\right), G^2\left(\left(1-\chi_n^{1,d}\right)\bar{n}^s\right)\right)$$

- Optimality condition:

$$\underbrace{\frac{\partial u}{\partial c^1}\frac{\partial G^1}{\partial n^{1,d}}}_{\text{Mg. Welfare Product 1}} = \underbrace{\frac{\partial u}{\partial c^2}\frac{\partial G^2}{\partial n^{2,d}}}_{\text{Mg. Welfare Product 2}}$$

  - Equalization of *marginal welfare products* of using factor

Planning Problem: Elastic Factor

$$\max_{\left\{\chi_n^{1,d}, n^s\right\}} u\left(G^1\left(\chi_n^{1,d} n^s\right), G^2\left(\left(1-\chi_n^{1,d}\right) n^s\right), n^s\right)$$

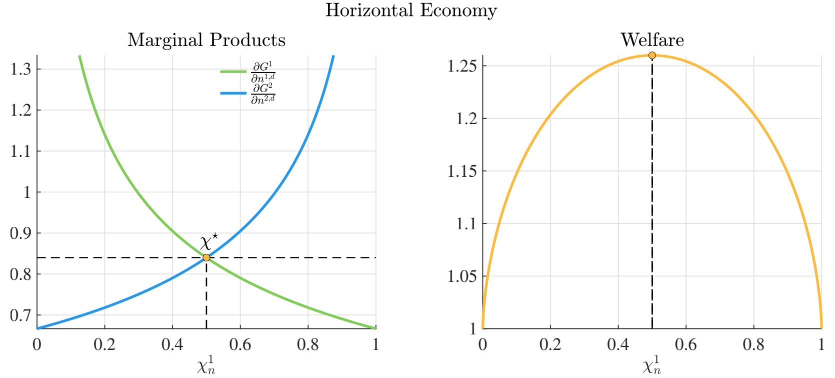
- Optimality conditions:

$$\underbrace{\frac{\partial u}{\partial c^1}\frac{\partial G^1}{\partial n^{1,d}}}_{\text{Mg. Welfare Product 1}} = \underbrace{\frac{\partial u}{\partial c^2}\frac{\partial G^2}{\partial n^{2,d}}}_{\text{Mg. Welfare Product 2}}$$

$$\underbrace{\chi_n^{1,d}\frac{\partial u}{\partial c^1}\frac{\partial G^1}{\partial n^{1,d}} + \left(1-\chi_n^{1,d}\right)\frac{\partial u}{\partial c^2}\frac{\partial G^2}{\partial n^{2,d}}}_{\substack{\text{Mg. Benefit of} \\ \text{increasing } n^s}} = \underbrace{-\frac{\partial u}{\partial n^s}}_{\substack{\text{Mg. Cost of} \\ \text{increasing } n^s}}$$

  - Cross-sectional factor use efficiency
  - Aggregate factor use efficiency (same as Robinson Crusoe!)

Horizontal Production



- Assumption: $u\left(c^1, c^2\right) = c^1 + c^2$

Competitive Equilibrium: Definition

- A *competitive equilibrium* is an allocation $\left\{c^1, c^2, y^1, y^2, n^{1,d}, n^{2,d}, n^s\right\}$, a price p, and a wage w, such that
  - i) individual chooses consumption and factor supply to maximize utility subject to the budget constraint taking prices as given

$$\max_{\left\{c^1, c^2, n^s\right\}} u\left(c^1, c^2, n^s\right) \quad \text{s.t.} \quad p^1 c^1 + p^2 c^2 = w n^s + \pi^1 + \pi^2,$$

  - i) each technology is operated to maximize profits choosing factor use taking prices as given

$$\pi^j = \max_{n^{j,d}} \left\{p^j y^j - w n^{j,d}\right\}, \quad \text{where} \quad y^j = G^j\left(n^{j,d}\right),$$

  - iii) and markets clear, that is, resource constraints hold:

$$\begin{aligned} y^j &= c^j && \text{Resource Constraint Goods} \\ n^s + \bar{n}^s &= n^d && \text{Resource Constraint Factor} \end{aligned}$$

Competitive Equilibrium: Characterization

1. Individual problem:
   $\max_{\{c^1,c^2,n^s\}} u\left(c^1, c^2, n^s\right) \quad \text{s.t.} \quad p^1c^1 + p^2c^2 = wn^s + \pi^1 + \pi^2$
   - Optimality:

$$\frac{\partial u}{\partial c^j} = \lambda p^j \Rightarrow \frac{\frac{\partial u}{\partial c^1}}{\frac{\partial u}{\partial c^2}} = \frac{p^1}{p^2} \quad \text{and} \quad \frac{\partial u}{\partial n^s} + w\lambda = 0$$

$$\Rightarrow \frac{-\frac{\partial u}{\partial n^s}}{\frac{\partial u}{\partial c^1}} = \frac{w}{p^1} \text{ and } \frac{-\frac{\partial u}{\partial n^s}}{\frac{\partial u}{\partial c^2}} = \frac{w}{p^2}$$

2. Technology: $\pi = \max_{n^d} \left\{py - wn^d\right\} \quad \text{where} \quad y = G\left(n^d\right)$
   - Optimality: $\rightarrow$ "mg. cost pricing"

$$\underbrace{p^j \frac{\partial G^j}{\partial n^{j,d}}}_{\substack{\text{Mg. Revenue} \\ \text{Product}}} = \underbrace{w}_{\substack{\text{Cost of} \\ \text{Factor (wage)}}} \implies \frac{\partial G^j}{\partial n^{j,d}} = \frac{w}{p^j}$$

   - Combining:

$$p^1 \frac{\partial G^1}{\partial n^{1,d}} = w = p^2 \frac{\partial G^2}{\partial n^{2,d}}$$

     - Equalization of *marginal revenue products*

Competitive Equilibrium: Characterization

- Combining both:

$$\frac{\partial u}{\partial c^1}\frac{\partial G^1}{\partial n^{1,d}} = \frac{\partial u}{\partial c^2}\frac{\partial G^2}{\partial n^{2,d}} = -\frac{\partial u}{\partial n^s}$$

  - Same as planning solution!
  - First Welfare Theorem: CE is Pareto Optimal

Remarks

1. *(Goods vs. Technologies)*: If two goods are perfect substitutes

$$u\left(c^1 + c^2, n^s\right)$$

   subject to

$$c^1 + c^2 = y^1 + y^2$$

   and

$$y^1 + y^2 = G^1\left(n^{1,d}\right) + G^2\left(n^{2,d}\right)$$

   - J should be thought of as "technology" rather than "good"
2. *Misallocation*: Cross-sectional factor efficiency
   - Are factors allocated efficiently?
   - Different from: is the aggregate amount of factor efficient?
   - Hsieh and Klenow (2009) and many others

Outline: Static Production Economies

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    i) Robinson Crusoe
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2. General Static Production Economies
    - Efficiency and Welfare
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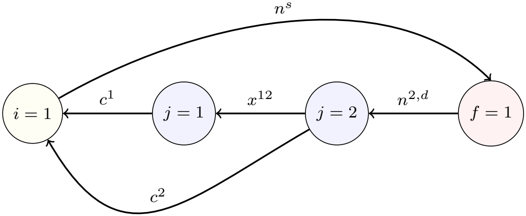
3. Applications

Vertical Production Economy: $I = F = 1$, $J = 2$

- (Third) Most elementary production model
  - $I = 1$ individual, indexed by $i \in \mathcal{I} = \{1\}$
  - $J > 1$ goods, indexed by $j \in \mathcal{J} = \{1, \dots, J\}$
  - $F = 1$ factor, indexed by $f \in \mathcal{F} = \{1\}$
- Factor can be fixed or elastically supplied



  - We could allow for $n^{1,d} > 0$
- Useful to think about Intermediate Goods
  - Supply chains, disaggregated production, etc.

Remark

1. *(Final vs mixed vs. intermediate goods)* A good is
   - final: consumed directly by individuals
   - intermediate: used to produced other goods
   - mixed: both are simultaneously true

- Goods can be
  i) pure final
  ii) pure intermediate
  iii) mixed
- In this economy: $j = 1$ is pure final, $j = 2$ is mixed
  - But if $c^2 = 0$, then $j = 2$ is pure intermediate

Vertical Production Economy: Physical Structure

- Preferences

$$V = u\left(\left\{c^j\right\}_{j\in\mathcal{J}}, n^s\right)$$

- Technologies

$$y^1 = G^1\left(x^{12}\right)$$
$$y^2 = G^1\left(n^d\right)$$

- Good resource constraints

$$y^1 = c^1$$
$$y^2 = c^2 + x^2$$

- Factor resource constraint

$$n^s + \bar{n}^s = n^d$$

- An *allocation* ($J = 2$) is $\left\{c^1, c^2, y^1, y^2, x^{12}, n^d, n^s\right\}$

Planning Problem

$$\max_{\{c^1,c^2,y^1,y^2,x^{12},n^d,n^s\}} u\left(c^1, c^2, n^s\right)$$

subject to

$$\begin{aligned}
y^1 &= G^1\left(x^{12}\right) && \text{Technology Good 1} \\
y^2 &= G^2\left(n^d\right) && \text{Technology Good 2} \\
y^1 &= c^1 && \text{Resource Constraint Good 1} \\
y^2 &= c^2 + x^{12} && \text{Resource Constraint Good 2} \\
n^s + \bar{n}^s &= n^d && \text{Resource Constraint Factor}
\end{aligned}$$

- Fixed factor: 1 degree of freedom
- Elastic factor: 2 degrees of freedom

Planning Problem: Fixed Factor

$$\max_{\{n^{1,d}, n^{2,d}\}} u\left(G^1\left(x^{12}\right), G^2\left(\bar{n}^s\right) - x^{12}\right)$$

- In terms of intermediate use shares: $\phi_x^2 = \frac{x^2}{y^2}$

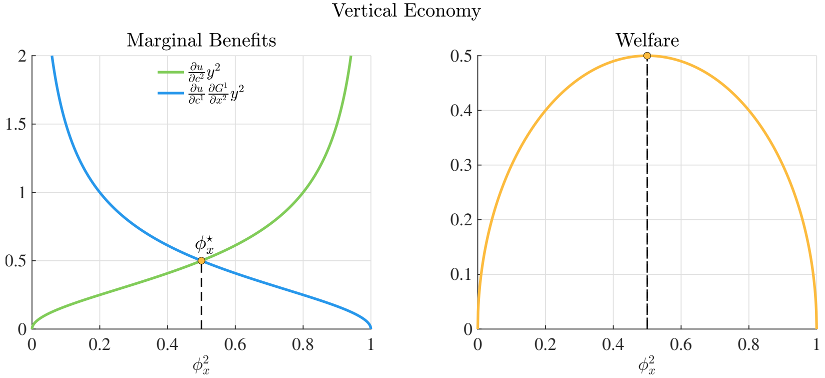
$$\max_{\phi_x^2} u\left(G^1\left(\phi_x^2 y^2\right), \left(1 - \phi_x^2\right) y^2\right)$$

- Optimality condition:

$$\underbrace{\frac{\partial u}{\partial c^1}\frac{\partial G^1}{\partial x^{12}} y^2}_{\text{Mg. Benefit of } x^{12}\uparrow} = \underbrace{\frac{\partial u}{\partial c^2} y^2}_{\text{Mg. Cost of } x^{12}\uparrow}$$

  - $MWP = MRS$
- Similar with elastic factor: “homework”

Vertical Production



Competitive Equilibrium: Definition

- A *competitive equilibrium* is an allocation $\left\{c^1, c^2, y^1, y^2, x^{12}, n^d, n^s\right\}$, a price p, and a wage w, such that
  - i) individual chooses consumption and factor supply to maximize utility subject to the budget constraint taking prices as given

$$\max_{\left\{c^1, c^2, n^s\right\}} u\left(c^1, c^2, n^s\right) \quad \text{s.t.} \quad p^1 c^1 + p^2 c^2 = w n^s + \pi^1 + \pi^2,$$

  - i) each technology is operated to maximize profits taking prices as given

$$\pi^1 = \max_{n^d} \left\{p^1 y^1 - p^2 x^{12}\right\}, \quad \text{where} \quad y^1 = G^1\left(x^{12}\right)$$

$$\pi^2 = \max_{n^d} \left\{p^2 y^2 - w n^d\right\}, \quad \text{where} \quad y^2 = G^2\left(n^d\right)$$

  - iii) and markets clear, that is, resource constraints hold:

$$y^1 = c^1 \qquad \text{Resource Constraint Good 1}$$

$$y^2 = c^2 + x^{12} \qquad \text{Resource Constraint Good 2}$$

$$n^s + \bar{n}^s = n^d \qquad \text{Resource Constraint Factor}$$

Competitive Equilibrium: Characterization

- Individual problem:
  $\max_{\{c^1,c^2,n^s\}} u\left(c^1,c^2,n^s\right) \quad \text{s.t.} \quad p^1c^1+p^2c^2=wn^s+\pi^1+\pi^2$
  - Optimality: (same as horizontal)

$$\frac{\partial u}{\partial c^j}=\lambda p^j \Rightarrow \frac{\frac{\partial u}{\partial c^1}}{\frac{\partial u}{\partial c^2}}=\frac{p^1}{p^2} \quad \text{and} \quad \frac{\partial u}{\partial n^s}+w\lambda=0$$

$$\Rightarrow \frac{-\frac{\partial u}{\partial n^s}}{\frac{\partial u}{\partial c^1}}=\frac{w}{p^1} \text{ and } \frac{-\frac{\partial u}{\partial n^s}}{\frac{\partial u}{\partial c^2}}=\frac{w}{p^2}$$

- Technology: Optimality $\rightarrow$ "mg. cost pricing"

$$p^2\frac{\partial G^2}{\partial n^d}=w \quad \Longrightarrow \quad \frac{\partial G^2}{\partial n^d}=\frac{w}{p^2}$$

$$p^1\frac{\partial G^1}{\partial x^{12}}=p^2 \quad \Longrightarrow \quad \frac{\partial G^1}{\partial x^{12}}=\frac{p^2}{p^1}$$

Competitive Equilibrium: Characterization

- Combining:

$$\frac{\partial u}{\partial c^1}\frac{\partial G^1}{\partial x^{12}} = \frac{\partial u}{\partial c^2}$$

  - good 2 efficiently allocated consumption vs. intermediate use
- Same as planning solution!
- First Welfare Theorem: CE is Pareto Optimal

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Quasilinear Economy

- Marshallian supply-demand diagram is a special GE economy
  - Partial Equilibrium
  - Quasilinear preferences $\rightarrow$ No income effects!
- Question: how many goods are in the supply-demand diagram?

Quasilinear Economy

- Marshallian supply-demand diagram is a special GE economy
  - Partial Equilibrium
  - Quasilinear preferences $\rightarrow$ No income effects!
- Question: how many goods are in the supply-demand diagram?
  - Two!
- Interpretation: the good considered is a small part of your budget
  Willig (1976)
- $I = 2$ individuals
- $J = 2$ goods
- $F = 1$ factor

Quasilinear Economy: Physical Structure

- Preferences: quasilinear in good 1 (same good!)

$$V^1 = c^{11} + \Psi^1\left(c^{12}\right)$$
$$V^2 = c^{21} + \Psi^2\left(n^s\right)$$

  - $\Psi^i(\cdot)$ is just a way of writing $u^i(\cdot)$
- Endowment of good 1 (assume very large so $c^{ij} > 0$)

$$\bar{y}^1 = c^1$$

  - where $\bar{y}^1 = \bar{y}^{11}$ and $c^1 = c^{11} + c^{21}$
- Technology for good 2

$$y^2 = G^2\left(n^d\right)$$

- Resource constraints

$$y^2 = c^2$$
$$n^s = n^d$$

  - where $c^2 = c^{12}$
- An *allocation* is $\left\{c^{11}, c^{12}, c^{21}, c^{22}, y^1, y^2, n^d, n^s\right\}$

Competitive Equilibrium: Definition

- A *competitive equilibrium* is an allocation $\left\{c^{11}, c^{12}, c^{21}, c^{22}, y^{1}, y^{2}, n^{d}, n^{s}\right\}$, a price p, and a wage w, such that

  i) individuals chooses consumption and factor supply to maximize utility subject to the budget constraint taking prices as given

$$\max_{\left\{c^{11},c^{12}\right\}} c^{11} + \Psi^{1}\left(c^{12}\right) \quad \text{s.t.} \quad c^{11} + p^{2}c^{12} = \bar{y}^{11} + \pi^{2}$$

$$\max_{\left\{c^{21},n^{s}\right\}} c^{21} + \Psi^{2}\left(n^{s}\right) \quad \text{s.t.} \quad c^{21} = w^{1}n^{s}$$

  i) technology is operated to maximize profits choosing factor use taking prices as given

$$\pi^{2} = \max\left\{p^{2}y^{2} - w^{1}n^{d}\right\}, \quad \text{where} \quad y^{2} = G^{2}\left(n^{d}\right)$$

  iii) and markets clear, that is, resource constraints hold:

$$y^{j} = c^{j} \qquad \text{Resource Constraint Goods}$$

$$n^{s} + \bar{n}^{s} = n^{d} \qquad \text{Resource Constraint Factor}$$

Competitive Equilibrium: Characterization

- Demand side:

$$\max \Psi^1\left(c^{12}\right) - p^2 c^{12} \quad \Longrightarrow \quad \boxed{\frac{\partial \Psi^1}{\partial c^{12}} = p^2} \Rightarrow c^{12}\left(p^2\right) \text{ Demand}$$

- Supply side:

$$\max w^1 n^{11,s} + \Psi^2\left(n^s\right) \quad \Longrightarrow \quad \boxed{w^1 = -\frac{\partial \Psi^2}{\partial n^s}} \Rightarrow n^s\left(w^1\right) \text{ Factor Supply}$$

- Profit maximization:

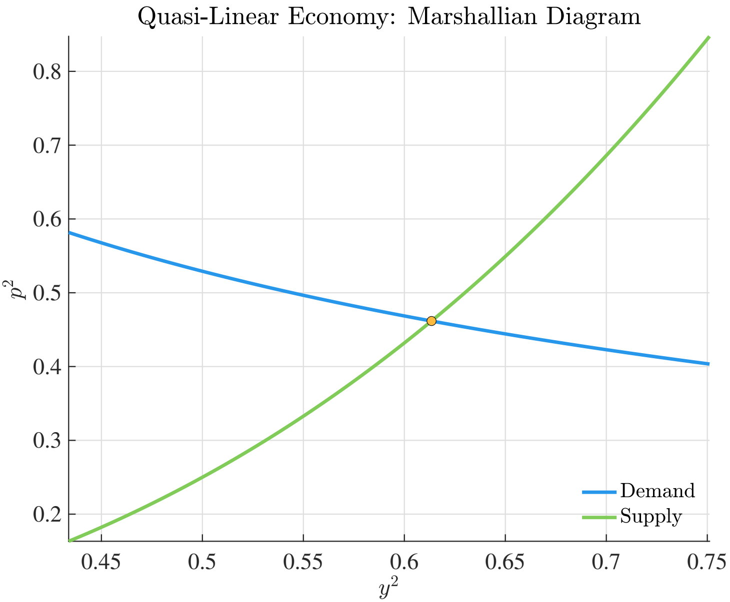
$$\boxed{p^2 \frac{\partial G^2}{\partial n^d} = w^1} \Rightarrow n^d\left(w^1, p^2\right) \text{ Factor Demand}$$

- Combine last two equations (and $c^{12} = y^2 = G^2\left(n^d\right)$) to find

$$\boxed{p^2 \frac{\partial G^2}{\partial n^d} = -\frac{\partial \Psi^2}{\partial n^s}} \Rightarrow c^{12}\left(p^2\right) \text{ Supply.}$$

- Supply + Demand $\rightarrow$ Marshallian Diagram
  - Lots of GE in the background!

Vertical Production



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