ECON 500a
General Equilibrium and Welfare Economics
Production Economies: Applications

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Outline: Static Production Economies

1. Elementary Static Production Economies
2. General Static Production Economy
3. Efficiency and Welfare
4. Applications

Application: Optimal Income Tax

- Foundational Optimal Taxation Problem
- How to optimally tax (and redistributive) labor income?
  - Mirrlees (1971) ⇒ Nonlinear tax
  - Sheshinski (1972) ⇒ Linear tax
    - Easier but similar insights
- Recent surveys: Piketty and Saez (2013) and Kaplow (2024)
- Basic tradeoff
  - Welfare gains from redistribution
  - Welfare/efficiency losses from production efficiency (aggregate factor efficiency)

Application: Optimal Income Tax

- $I = F = 2,\ J = 1$
- Individuals $i \in \{1, 2\}$ have identical preferences given by

$$V^i = u\left(c^i, \left\{n^{if,s}\right\}_f\right)$$

- Assumption: $i = 1$ can only supply factor $f = 1$
  - Why? We want individuals to have different productivity and to receive different wages
  - Alternative assumption: different preferences $u(\cdot)$
- So we write

$$V^i = u\left(c^i, n^i\right)$$

- Subject to

$$c^i = (1 - \tau)\, w^i n^i + g, \quad \text{where} \quad g = \frac{1}{I}\tau \sum_i w^i n^i$$

  - g is a "demogrant": tax revenues shared equally
- Linear technology to produce the consumed good
  - $\pi = \left(a^1 - w^1\right) n^1 + \left(a^2 - w^2\right) n^2$ (assume interior)
  - Wages earned by each individual are effectively "primitives"
  - No profits

Application: Optimal Income Tax

- Functional form: $u\left(c,n\right)=\frac{1}{1-\gamma}\left(c-\alpha\frac{n^{\sigma}}{\sigma}\right)^{1-\gamma}$
  - Quasilinear kills income effects! $\rightarrow$ easy
- Optimality condition

$$\left(1-\tau\right)w^{i}\frac{\partial u}{\partial c^{i}}+\frac{\partial u}{\partial n^{i}}=0\rightarrow n^{i}\left(\tau\right)=\left(\frac{\left(1-\tau\right)w^{i}}{\alpha}\right)^{\frac{1}{\sigma-1}}$$

- Closed form for $g\left(\tau\right)=\tau\frac{1}{I}\sum_{i}w^{i}n^{i}\left(\tau\right)$
  - With income effects: $n^{i}\left(\tau;g\right)\rightarrow$ we would have a fixed point

Application: Optimal Income Tax

- Tax perturbation, with g adjusting:

$$\frac{dV^i}{d\tau} = \frac{\partial u}{\partial c^i}\frac{dc^i}{d\tau} + \frac{\partial u}{\partial n^i}\frac{dn^i}{d\tau}$$

$$= \frac{\partial u}{\partial c^i}\left(-w^i n^i + \frac{dg}{d\tau}\right) + \underbrace{\left(\frac{\partial u}{\partial c^i}(1-\tau)\,w^i + \frac{\partial u}{\partial n^i}\right)}_{=0}\frac{dn^i}{d\tau}$$

- Normalized welfare gains given by

$$\frac{dV^{i|\lambda}}{d\tau} = \frac{\frac{dV^i}{d\tau}}{\lambda^i} = -w^i n^i + \frac{dg}{d\tau}$$

  - $\lambda^i = \frac{\partial u}{\partial c^i}$ and $\frac{dg}{d\tau} = \frac{1}{I}\left(\sum_i w^i n^i + \tau \sum_i w^i \frac{dn^i}{d\tau}\right)$

- Efficiency/redistribution decomposition:

$$\boxed{\frac{dW^\lambda}{d\tau} = \underbrace{-\tau \sum_i w^i \left(-\frac{dn^i}{d\tau}\right)}_{\Xi^E \text{ (Efficiency)}} + \underbrace{\mathbb{C}ov_i^\Sigma\left[\omega^i, -w^i n^i\right]}_{\Xi^{RD} \text{ (Redistribution)}}}$$

- Individual weights: $\omega^i = \frac{\alpha^i \frac{\partial u}{\partial c^i}}{\frac{1}{I}\sum_i \alpha^i \frac{\partial u}{\partial c^i}}$

Application: Optimal Income Tax

- Note that (using FOC)

$$\tau w^i = \underbrace{w^i}_{=MWP^f} - \underbrace{\left(\frac{-\frac{\partial u}{\partial n^i}}{\frac{\partial u}{\partial c^i}} \right)}_{MRS^f}$$

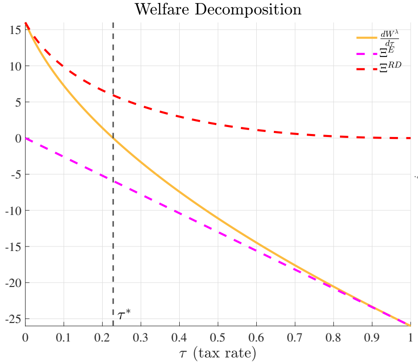
- So

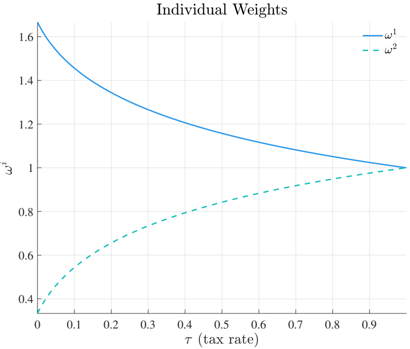
$$\Xi^E = -\tau \sum_i w^i \left(-\frac{dn^i}{d\tau} \right) = \sum_f \underbrace{\left(MWP^f - MRS^f \right)}_{>0} \underbrace{\frac{dn^f}{d\tau}}_{<0}$$

  - What is the cost of the tax → both individuals work too little
- Efficiency losses from

production efficiency → aggregate factor efficiency

Application: Optimal Income Tax





Application: Optimal Income Tax

- Optimal Tax Formula: $\frac{dW^{\lambda}}{d\tau} = 0$

$$\boxed{\tau^{\star} = \frac{\mathbb{C}ov_i^{\Sigma}\left[\omega^i, -w^i n^i\right]}{\sum_i w^i n^i \left(-\frac{d\log n^i}{d\tau}\right)}}$$

- All elements are endogenous to the tax → tricky
- But maybe you can measure them → sufficient statistics

Application: Hulten's Theorem

- In competitive economies:
  - Exchange (consumption and factor supply) efficiency = 0
  - Cross-sectional factor efficiency = 0
  - Aggregate factor efficiency = 0
- What is left?

$$\Xi^{AE,P} = \underbrace{\sum_j AMRS_c^j \frac{\partial G^j}{\partial \theta}}_{\substack{\text{Technology}\\\text{Change}}} + \underbrace{\sum_j AMRS_c^j \frac{d\bar{y}^{j,s}}{d\theta}}_{\substack{\text{Good Endowment}\\\text{Change}}} + \underbrace{\sum_f AMWP_n^f \frac{d\bar{n}^{f,s}}{d\theta}}_{\substack{\text{Factor Endowment}\\\text{Change}}}$$

- Economic question: what is the welfare/efficiency impact of
  1. Marginal change in technology ⇒ Hulten's Theorem (Hulten, 1978)
     Popularized by Gabaix (2011)
  2. Marginal unit of a good
  3. Marginal unit of a factor
- **Applications**: AI, Robots, Automation, etc...
  Acemoglu (2024): The Simple Macroeconomics of AI

Application: Hulten's Theorem

- In competitive economies:

$$\Xi^{AE,P} = \underbrace{\sum_j p^j \frac{\partial G^j}{\partial \theta}}_{\substack{\text{Technology} \\ \text{Change}}} + \underbrace{\sum_j p^j \frac{d\bar{y}^{j,s}}{d\theta}}_{\substack{\text{Good Endowment} \\ \text{Change}}} + \underbrace{\sum_f w^f \frac{d\bar{n}^{f,s}}{d\theta}}_{\substack{\text{Factor Endowment} \\ \text{Change}}}$$

Theorem

(Hulten's Theorem) In competitive economies, the efficiency impact of a marginal change in the technology of good j is given by its price p^j.

Application: Hulten's Theorem

- In competitive economies:

$$\Xi^{AE,P} = \underbrace{\sum_j p^j \frac{\partial G^j}{\partial \theta}}_{\substack{\text{Technology} \\ \text{Change}}} + \underbrace{\sum_j p^j \frac{d\bar{y}^{j,s}}{d\theta}}_{\substack{\text{Good Endowment} \\ \text{Change}}} + \underbrace{\sum_f w^f \frac{d\bar{n}^{f,s}}{d\theta}}_{\substack{\text{Factor Endowment} \\ \text{Change}}}$$

Theorem

(Hulten's Theorem) In competitive economies, the efficiency impact of a marginal change in the technology of good j is given by its price p^j.

- Proof: In a competitive economy: $AMRS_c^j = MRS_c^{ij} = \frac{\frac{\partial u^i}{\partial c^{ij}}}{\lambda^i} = p^j$
- Same answer for marginal unit of a good: $AMRS_c^j = p^j$
- Similar answer for marginal unit of a factor
  - Proof: $AMWP_n^f = MWP_n^{jf} = MRS_n^{if,s} = \frac{\frac{\partial u^i}{\partial n^{if,s}}}{\lambda^i} = w^f$
- **Economic insight**: it is good to have more goods with high prices and factors with high wages!
  - We should clone Lebron James
  - Note: marginal result → the 100th LJ is less valuable

Application: Hulten's Theorem

- **Remark 1**: Hulten's theorem typically stated as "Domar weights":

$$\frac{p^j y^j}{\sum_j p^j c^j}$$

- This is because it considers
  1. proportional technology shocks (and no endowments)
     - $y^j = \theta \tilde{G}^j = e^{\log\theta} \theta \tilde{G}$, so $AMRS_c^j \frac{\partial G^j}{\partial \log\theta} = p^j y^j$
  2. $\sum_j p^j c^j$ as welfare numeraire ($\sim$ "GDP")
     - This may be natural when $I = 1$ and homothetic preferences
- **Remark 2**: Hulten's theorem at times stated as applying to *efficient* economies $\Rightarrow$ Not quite

  Check Dávila and Schaab (2024) for details
  - Hulten's Theorem applies to
    i) (frictionless) competitive economies
    ii) interior (with non-negative constraints not binding) efficient economies (with $AMRS_c^j$ playing the role of the price)

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