ECON 500a
General Equilibrium and Welfare Economics
Efficiency and Welfare: Dynamic Stochastic Economies

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Updated: December 05, 2024

Outline: Dynamic Stochastic Economies

1. Dynamic Economics
2. Stochastic Economics
3. Asset Pricing
4. Efficiency and Welfare
5. Incomplete Markets
6. Production, Firms, Ownership
7. Accumulation Technologies

- Readings
  - MWG: Chapter 19.I

Roadmap

1. Constrained Inefficiency with Incomplete Markets
2. ~~Welfare Assessments~~
3. ~~Welfare with Heterogeneous Beliefs~~
4. ~~OLG~~
5. ~~Applications~~
6. Financial Innovation

Constrained Inefficiency with Incomplete Markets

- Complete Markets ⇒ Welfare Theorems apply
  - Follows trivially from consolidation arguments
  - All proofs of First Welfare Theorem apply unchanged
- Incomplete Markets ⇒ Obviously there are Pareto improvements
  - Planning solution is not achieved ⇒ This is "cheating"
  - Planner has more power than the agents ("chicken model")
    - Planner is not constrained by asset span
- Can we find Pareto improvements <u>respecting the asset span</u>?
  - *Constrained Pareto Improvements*
- History
  - Diamond (1967): constrained efficient incomplete markets model
  - Hart (1975): first example of constrained inefficiency
  - Geanakoplos and Polemarchakis (1986): proof of generic constrained inefficiency
  - Dávila and Korinek (2018): distributive vs. collateral/frictional externalities

Hart 75 Inefficiency Example

- $I = 2$, $T = 1$, and $J > 1$ with no financial markets ($Z = 0$)
  - Two Edgeworth box economies
- Preferences: $V^i = u^i\left(\left\{c_0^{ij}\right\}_{j\in\mathcal{J}}\right) + \beta^i u^i\left(\left\{c_1^{ij}\right\}_{j\in\mathcal{J}}\right)$
- Examples: suppose two "static" equilibria at each date:

$$\{A, B\} \quad \text{and} \quad \{C, D\}$$

  There are β^i such that a (dynamic) competitive equilibrium dominates another
- Flow utilities are $\{2, 4\}$ in A, $\{6, 2\}$ in B, $\{5, 6\}$ in C, and $\{8, 2\}$ in D

Equilibrium	V^1	V^2
(A, C)	$2 + \beta^1 5$	$4 + \beta^2 6$
(A, D)	$2 + \beta^1 8$	$4 + \beta^2 2$
(B, C)	$6 + \beta^1 5$	$2 + \beta^2 6$
(B, D)	$6 + \beta^1 8$	$2 + \beta^2 2$

$$\underbrace{\Longrightarrow}_{\beta^1, \beta^2 \to 1}$$

	V^1	V^2
(A, C)	7	10
(A, D)	10	6
(B, C)	11	8
(B, D)	14	4

- Can you find Pareto dominated equilibria? What if $\beta^1, \beta^2 \to 1$?
  - Equilibrium (A, D) is Pareto dominated by (B, C)

Constrained Inefficiency with Incomplete Markets

- Problem with Hart (1975) example $\Rightarrow$ multiple equilibria
- Proof of constrained inefficiency in the simplest incomplete market economy
  - Multi-good perfect foresight economy with no financial markets
  - $I > 1$, $T = 1$, and $J > 1$ with no financial markets ($Z = 0$)
  - Two repeated static exchange economies
  - Version of Proof #3 of First Welfare Theorem in Block I
- Individuals solve

$$\max_{\left\{c_t^{ij}\right\}} \sum_t \left(\beta^i\right)^t u^i \left(\left\{ c_t^{ij} \right\}_{j \in \mathcal{J}} \right) \quad \text{s.t.} \quad \sum_j p_t^j c_t^{ij} = \sum_j p_t^j \bar{y}_t^{ij}, \ \forall t, \ \forall i.$$

- Are markets complete?
  - $S = 1$ and $Z = 0$, so $S > Z$
- How many budget constraint?

Proof I

- Starting from a competitive equilibrium allocation, individual welfare gains of a perturbation are

$$\frac{dV^i}{d\theta} = \sum_t \left(\beta^i\right)^t \sum_j \frac{\partial u^i}{\partial c_t^{ij}} \frac{dc_t^{ij}}{d\theta} = \lambda_0^i \sum_t \left(\beta^i\right)^t \sum_j \frac{\frac{\partial u^i}{\partial c_t^{ij}}}{\lambda_0^i} \frac{dc_t^{ij}}{d\theta}$$

- Without loss, we use date-0 dollars as welfare numeraire, so λ_0^i denotes the Lagrange multiplier in individual i's date 0 budget constraint
- Since any perturbation that modifies individual demands must satisfy individual budget constraints, it must be that at both dates $t = \{0, 1\}$:

$$\sum_j p_t^j \frac{dc^{ij}}{d\theta} = \sum_j \frac{dp_t^j}{d\theta} \left(\bar{y}_t^{ij} - c_t^{ij}\right)$$

Proof II

- Aggregate changes in the market value of individual i's consumption at date t, $\sum_j p^j \frac{dc_t^{ij}}{d\theta}$, need to equal the sum of the distributive pecuniary effects — using the language of Dávila and Korinek (2018) — of the perturbation, $\sum_j \frac{dp^j}{d\theta}\left(\bar{y}^{ij} - c^{ij}\right)$.
- These pecuniary effects are composed by
  i) net trading positions (net buying or net selling), $\bar{y}^{ij} - c^{ij}$, and
  ii) sensitivity of equilibrium prices, $\frac{dp^j}{d\theta}$.
- Therefore, using individual optimality conditions:
  - $\frac{\partial u^i}{\partial c_0^i} = \lambda_0^i p_0^j$ and $\beta^i \frac{\partial u^i}{\partial c_1^i} = \lambda_1^i p_1^j$

$$\frac{\frac{dV^i}{d\theta}}{\lambda_0^i} = \sum_j p_0^j \frac{dc_0^{ij}}{d\theta} + \beta^i \frac{\lambda_1^i}{\lambda_0^i} \sum_j p_1^j \frac{dc_1^{ij}}{d\theta}$$

$$= \sum_j \frac{dp_0^j}{d\theta}\left(\bar{y}_0^{ij} - c_0^{ij}\right) + \beta^i \frac{\lambda_1^i}{\lambda_0^i} \sum_j \frac{dp_1^j}{d\theta}\left(\bar{y}_1^{ij} - c_1^{ij}\right)$$

Proof III

- Aggregating across all individuals and exploiting market clearing:

$$\sum_i \frac{\frac{dV^i}{d\theta}}{\lambda^i} = \sum_j \frac{dp_0^j}{d\theta} \underbrace{\sum_i \left(\bar{y}_0^{ij} - c_0^{ij}\right)}_{=0} + \beta^i \sum_i \frac{\lambda_1^i}{\lambda_0^i} \sum_j \frac{dp_1^j}{d\theta} \left(\bar{y}_1^{ij} - c_1^{ij}\right)$$

$$= \beta^i \sum_i \frac{\lambda_1^i}{\lambda_0^i} \sum_j \frac{dp_1^j}{d\theta} \left(\bar{y}_1^{ij} - c_1^{ij}\right) = \mathbb{C}ov_i \left[\frac{\lambda_1^i}{\lambda_0^i}, \sum_j \frac{dp_1^j}{d\theta} \left(\bar{y}_1^{ij} - c_1^{ij}\right)\right] \neq 0$$

- Distributive pecuniary effects *cancel out* (add up to zero) in the aggregate *at each date*
  - But the welfare impact of a perturbation is in general not zero in the aggregate
- Possible to find perturbations that make all individuals better off
  - Competitive equilibrium is *constrained Pareto inefficient*

Intuition

- Individuals have different valuations for consumption at dates 0 and 1.
  - Shadow valuations/interest rates $\frac{\lambda_1^i}{\lambda_0^i}$ are not equalized.
  - No reason for them to be equalized $\Rightarrow$ No markets!
- How to construct Pareto improvement?
  - With $I = 2$, let's change demands so that the distributive pecuniary effects at date 1, given by $\sum_j \frac{dp_1^j}{d\theta}\left(\bar{y}_1^{ij} - c_1^{ij}\right)$, benefit the individual who prefers to consume at date (with a high $\frac{\lambda_1^i}{\lambda_0^i}$)
- Then ensure that the distributive pecuniary effects at date 0 compensate the other individual
- Generic constrained inefficiency result of Geanakoplos and Polemarchakis (1986) has a rank condition
  - We need sufficiently many dates, states, goods, or assets relatively to the number of individuals

$$\underbrace{T + S + J + Z}_{\text{dates + states + goods + assets}} \gg \underbrace{I}_{\text{individuals}}$$

Constrained Efficient Incomplete Market Economies

- Constrained efficient economies
  i) $T = 0$, $I \geq 1$: Static Economies
  ii) $I = 1$: Single Individual/Representative Agent Economies
  iii) $T = 1$, $S \geq 1$, $J = 1$, $I \geq 1$: Two-Date Single-Good Finance Economies (e.g. CAPM)
  iv) $T = 1$, $S = 1$, $J > 1$, $I \geq 1$: Two-Date Multiple-Good Deterministic Economies

Constrained Inefficient Incomplete Market Economies?

- (Generically) Constrained inefficient economies
  i) $T = 1,\ S = 1,\ J > 1,\ I = 2$: Two-Date Multiple-Good Deterministic Economies
  ii) $T = 1,\ S > 1,\ J > 1,\ I \geq 2$: Two-Date Multiple-Good Finance Economies ($S \gg I$)
  iii) $T > 1,\ S = 1,\ J = 1,\ I \geq 1$: Deterministic Single-Good Economies ($T \gg I$)
  iv) $T > 1,\ S \geq 1,\ J = 1,\ I \geq 1$: Dynamic Stochastic Single-Good Economies ($T + S + Z \gg I$)
  v) $T > 1,\ S \geq 1,\ J = 1,\ I \geq 1$: Dynamic Stochastic Multi-Good Economies ($T + S + J + Z \gg I$)
- $J > 1$ or $T > 1$ are necessary to find constrained Pareto improvements

Prices Outside Budget Constraints

- Prices outside of budget constraint $\Rightarrow$ distinct source of constrained inefficiency
  - Collateral/Frictional externalities
- Example: borrowing constraint $b_t^i \leq q_t k_t^i$
- Closer to standard externalities

$$\tilde{u}^i\left(c_t^i; q_t\right) = u\left(c_t^i\right) + \eta_t^i\left(b_t^i - q_t k_t^i\right)$$

  - Agents do not internalize that they directly affect the choice set of others
- Check Dávila and Korinek (2018) for more

Wrapping Up

- Pecuniary externalities are pervasive
  - Every Walrasian model has *distributive* pecuniary externalities
  - They add-up to zero under complete markets
  - They allow for welfare-improving price induced redistribution under incomplete markets
- Prices that appear anywhere else (e.g. collateral constraints, incentive constraints, etc) also generate externalities
- **Main takeaway**: Walrasian complete markets models are very special
  - Price-taking agents only interact through linear prices that only enter budget constraints
  - Prices only reflect scarcity
  - Agents have perfect insurance, so all decisions are linked

Financial Innovation

- Questions
  - Is it better to have more (financial) markets?
  - Which financial markets should be created?
  - Shiller (1993): Macro Markets
- Hart (1975) shows that introducing a new asset can make everyone worse off
  - Nice example of second-best economics: moving towards first-best does not guarantee a welfare improvement
- Technical challenge
  - It is hard to study the effect of introducing new assets
  - e.g., change asset span from 1 to 2 assets (lack of differentiability)
- Solution: differentiable perturbation $\Rightarrow$ relaxing a borrowing constraint

Financial Innovation: Environment

- $T = 1$, $I \geq 1$
- Preferences

$$V^i = u\left(c_0^i\right) + \beta \sum_s \pi(s)\, u\left(c_1^i(s)\right)$$

- Budget constraints

$$c_0^i = n_0^i + q_0 b_0^i$$
$$c_1^i(s) = n_1^i(s) - b_0^i,\ \forall s$$
$$b_0^i \leq \bar{b}$$

- Only risk-free asset is traded, agents face borrowing constraint
  - If $b_0^i > 0$ agent i borrows, if $b_0^i < 0$, agent i saves
- Equilibrium definition: consumption and debt allocations c_0^i, $c_1^i(s)$, and b_0^i, and a price for the risk-free asset q_0, such that agents solve their individual problems given a price q_0, and the risk-free bond market clears: $\sum_i b_0^i = 0$.

Financial Innovation: Solution

- Lagrangian

$$\mathcal{L}^i = u\left(c_0^i\right) + \beta \sum_s \pi(s)\, u\left(c_1^i(s)\right) - \lambda_0^i \left(c_0^i - n_0^i - q_0 b_0^i\right)$$
$$- \sum_s \lambda_1^i(s) \left(c_1^i(s) - n_1^i(s) + b_0^i\right) - \mu^i \left(b_0^i - \overline{b}\right)$$

- Borrowing FOC:

$$q_0 u'\left(c_0^i\right) - \beta \sum_s \pi(s)\, u'\left(c_1^i(s)\right) = \mu^i$$

- Note that $\left(b_0^i - \overline{b}\right)\mu^i = 0$
  - If $\mu^i = 0$, then standard Euler equations holds
  - If $\mu^i > 0$, then $b_0^i = \overline{b}$ and the FOC just pins down μ^i
- Let's focus on how varying $\overline{b}$ changes welfare

Financial Innovation: Individual Welfare Impact

- Financial innovation as relaxing borrowing constraints

$$\frac{dV^i}{d\bar{b}} = \underbrace{\overbrace{\left(u'\left(c_0^i\right)q_0 - \beta\sum_s \pi\left(s\right)u'\left(c_1^i\left(s\right)\right)\right)}^{=\mu^i\geq 0}\overbrace{\frac{db_0^i}{d\bar{b}}}^{\geq 0}}_{\text{Portfolio Effect}} + \underbrace{u'\left(c_0^i\right)\frac{dq_0}{d\bar{b}}b_0^i}_{\text{Distributive Pecuniary Effec}}$$

- Distributive pecuniary effects are zero-sum: $\sum_i \frac{dq_0}{d\bar{b}}b_0^i = 0$
- Portfolio effect is weakly positive:
  - $\mu^i > 0$ if $\frac{db_0^i}{d\bar{b}} = 1 > 0$
- One would expect that $\frac{dq_0}{d\bar{b}} < 0$
  - Higher demand for borrowing, high interest rates (lower q_0)
- Can relaxing a constraint make constrained agents worse off if prices are fixed? No
- Can relaxing a constraint make constrained agents worse off in General Equilibrium? Yes
- Can relaxing a constraint make all agents worse off? Yes, but not in this simple model (see next slide)

Financial Innovation: Aggregate Welfare Impact

- Normalized welfare gains for i:

$$\frac{\frac{dV^i}{d\bar{b}}}{u'\left(c_0^i\right)} = \underbrace{\frac{\mu^i}{u'\left(c_0^i\right)}\frac{db_1^i}{d\bar{b}}}_{\text{Portfolio}} + \underbrace{\frac{dq_0}{d\bar{b}}b_1^i}_{\text{Pecuniary}}$$

- Aggregating:

$$\Xi^E = \sum_i \frac{\frac{dV^i}{d\bar{b}}}{u'\left(c_0^i\right)} = \sum_i \frac{\mu^i}{u'\left(c_0^i\right)}\frac{db_1^i}{d\bar{b}} \geq 0$$

- Since the sum is positive, at least one agent has to be better off!
- How should we modify the model to find a case in which everyone is worse off?
  - What if we add another period? (assume perfect foresight afterwards)

Financial Innovation: Extended Model

- Welfare change for agent i: (assuming that we relax constraint in all periods)

$$\frac{dV^i}{d\bar{b}} = \underbrace{\overbrace{\left(u'\left(c_0^i\right)q_0 - \beta\sum_s \pi\left(s\right)u'\left(c_1^i\left(s\right)\right)\right)}^{\mu_0^i}\frac{db_0^i}{d\bar{b}}}_{\text{Portfolio Effect}}$$

$$+\underbrace{\sum_s \overbrace{\left(u'\left(c_1^i\left(s\right)\right)q_1\left(s\right) - \beta\sum_s \pi\left(s\right)u'\left(c_2^i\left(s\right)\right)\right)}^{\mu_1^i(s)}\frac{db_1^i\left(s\right)}{d\bar{b}}}_{\text{Portfolio Effect}}$$

$$\underbrace{+u'\left(c_0^i\right)\frac{dq_0}{d\bar{b}}b_0^i + \beta\sum_s \pi\left(s\right)u'\left(c_1^i\left(s\right)\right)\frac{dq_1\left(s\right)}{d\bar{b}}b_1^i\left(s\right)}_{\text{Pecuniary Effects}}$$

Financial Innovation: Everyone Worse Off

- Aggregated:

$$\sum_i \frac{\frac{dV^i}{d\bar{b}}}{u'\left(c_0^i\right)} = \overbrace{\sum_i \left(\frac{\mu_0^i}{u'\left(c_0^i\right)} \frac{db_1^i}{d\bar{b}} + \sum_s \frac{\mu_1^i\left(s\right)}{u'\left(c_0^i\right)} \frac{db_2^i\left(s\right)}{d\bar{b}} \right)}^{\geq 0} + \underbrace{\sum_i \left(\sum_s \frac{\beta u'\left(c_1^i\left(s\right)\right)}{u'\left(c_0^i\right)} \frac{dq_1\left(s\right)}{d\bar{b}} b_2^i\left(s\right) \right)}_{\gtreqless 0} \gtreqless 0$$

- What if $\frac{\beta u'\left(c_1^i(s)\right)}{u'\left(c_0^i\right)}$ is the same $\forall i$? How do we call this case?
  - In that case, $\sum_i \frac{\frac{dV^i}{d\bar{b}}}{u'\left(c_0^i\right)} \geq 0$!
- This shows that an increase in borrowing constraints starting from the first-best complete markets outcome is welfare decreasing on aggregate.
- Alternatively, relaxing borrowing constraints all the way to the first best (locally) is welfare improving on aggregate.

Financial Innovation: Final Remarks

1. We need S and T large relative to I to find a case in which everyone is worse off
2. We could have relaxed the constraint in only one period or state, what matters is that prices react to the change in the constraint (pecuniary effects)
3. Nothing hinges on the single-bond assumption, it's trivial to extend the results to many assets, as long as MRS across dates/states are not equalized
4. Full asset spanning + binding constraints on asset holdings $\Rightarrow$ Incomplete Markets
   - What really matters is that MRS are not equalized

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